

# *The MSSM with large $\tan\beta$ beyond the decoupling limit*

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# The Parameter $\tan \beta$

- ▶ MSSM contains two Higgs-doublets  $H_u$ ,  $H_d$ :

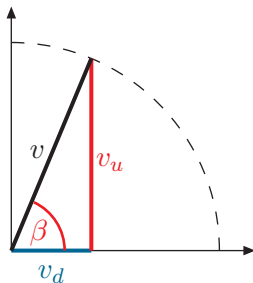
$$\mathcal{L}_{q,y} = -y_u^{ij} \bar{u}_i Q_j H_u - y_d^{ij} \bar{d}_i Q_j H_d + h.c.$$

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- ▶ VEVs:  $\langle H_u \rangle = v_u$ ,  $\langle H_d \rangle = v_d$   
with  $v_u^2 + v_d^2 = v^2 \approx (174 \text{ GeV})^2$   
 $\Rightarrow$  free parameter  $\tan \beta = v_u/v_d$
- ▶ large  $\tan \beta \Leftrightarrow$  small  $v_d$



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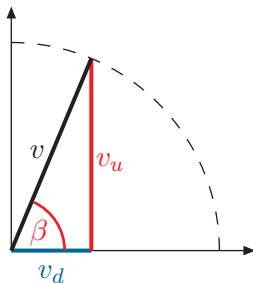
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- ▶ large  $\tan \beta \Leftrightarrow$  small  $v_d$

- ▶ Motivation for large  $\tan \beta \sim 50$ : Unification of  $y_t$  and  $y_b$  possible ( $\Rightarrow$  SO(10) GUTs)

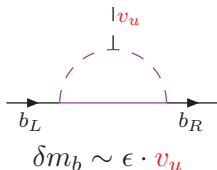
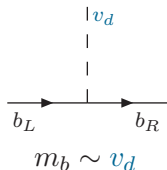


# The pattern of $\tan\beta$ -enhancement

- ▶ Consider tree-level amplitude with suppression  $v_d$ .  
One-loop corrections may involve  $v_u$  instead.

[Hall,Rattazzi,Sarid; Blazek,Raby,Pokorski]

- ▶ Example:  $b$ -quark mass



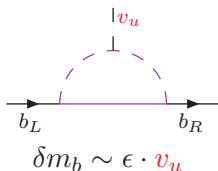
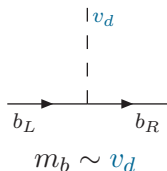
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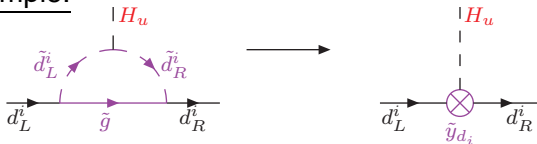


$$\frac{\delta m_b}{m_b} \sim \epsilon \cdot \tan\beta$$
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- ▶ Two possibilities to deal with such  $\mathcal{O}(1)$  corrections
  1. Effective Lagrangian for  $M_{\text{SUSY}} \gg v, M_{A^0}, M_{H^0}, M_{H^\pm}$
  2. Calculation in the full MSSM **beyond decoupling**

# Effective Lagrangian in the decoupling limit

- ▶ Integrate out all particles with masses  $M_{\text{SUSY}} \gg v$ , keep only SM particles and Higgs fields
- ▶ Example:

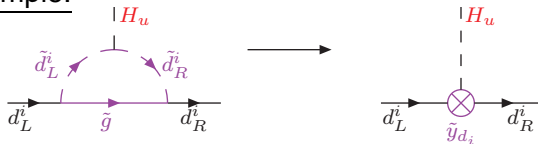


$$\mathcal{L}_{d,y}^{\text{eff}} = -y_{d_i} \bar{d}_i Q_i H_d - \tilde{y}_{d_i} \bar{d}_i Q_i H_u$$

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$$\mathcal{L}_{d,y}^{eff} = -y_{d_i} \bar{d}_i Q_i H_d - \tilde{y}_{d_i} \bar{d}_i Q_i H_u$$

- ▶ Consequence: Modified relation between  $y_{d_i}$  and  $m_{d_i}$

$$m_{d_i} = y_{d_i} v_d + \tilde{y}_{d_i} v_u$$

$\Rightarrow$

$$y_{d_i} = \frac{m_{d_i}}{v_d(1 + \epsilon_i \tan \beta)}$$

contains contributions of the form  $(\epsilon \tan \beta)^n$  to all orders  
 $\rightarrow$  **resummation?**

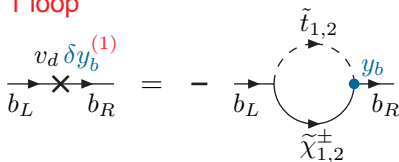


# Resummation beyond the decoupling limit

- ▶ Subtract  $\tan \beta$ -enhanced corrections to all orders by appropriate finite counterterms. [Carena,Garcia,Nierste,Wagner]

▶ Example:  $\Sigma_{b,\tilde{\chi}^\pm}^{RL}(y_b) = y_b v_d \Delta_b^{\tilde{\chi}^\pm}$ ,  $\Delta_b^{\tilde{\chi}^\pm} = \epsilon_b^{\tilde{\chi}^\pm} \tan \beta$

1 loop



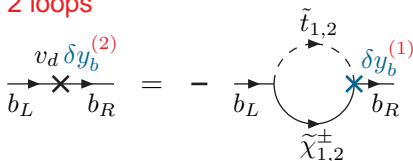
$$\begin{aligned} \delta y_b^{(1)} &= -\Delta_b^{\tilde{\chi}^\pm} y_b \\ &= -\Delta_b^{\tilde{\chi}^\pm} \frac{m_b}{v_d} \end{aligned}$$

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2 loops



$$\begin{aligned} \delta y_b^{(2)} &= -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(1)} \\ &= (-\Delta_b^{\tilde{\chi}^\pm})^2 \frac{m_b}{v_d} \end{aligned}$$

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n loops

$$\begin{aligned}
 \text{Tree-level vertex correction} &= - \text{Self-energy correction} \\
 \delta y_b^{(n)} &= -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)} \\
 &= (-\Delta_b^{\tilde{\chi}^\pm})^n \frac{m_b}{v_d}
 \end{aligned}$$

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n loops

$$\begin{aligned} \text{Tree-level diagram} &= - \text{Loop diagram} \\ \delta y_b^{(n)} &= -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)} \\ &= (-\Delta_b^{\tilde{\chi}^\pm})^n \frac{m_b}{v_d} \end{aligned}$$

$$y_b^0 = \frac{m_b}{v_d} \left( 1 - \Delta_b^{\tilde{\chi}^\pm} + \Delta_b^{\tilde{\chi}^\pm 2} - \dots \right) = \frac{m_b}{v_d (1 + \Delta_b^{\tilde{\chi}^\pm})}$$

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$$\delta y_b^{(n)} = -\Delta_b^{\tilde{\chi}^\pm} \delta y_b^{(n-1)}$$

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- ▶ **Explicit resummation** of contributions of the form

$$\Delta_b = \epsilon_b \tan \beta$$

## Why go beyond decoupling limit?

- ▶  $M_{\text{SUSY}} \sim v$  is natural.
  - ▶  $M_{\text{SUSY}} \gg v$  introduces hierarchy problem.
  - ▶ Scenarios with neutralino LSP involve several SUSY-particles with masses  $\sim v$ .
- ▶ Test accuracy of calculations done with the effective Lagrangian approach.
- ▶ Study  $\tan\beta$ -enhanced effects in couplings of SUSY-particles like  $\tilde{g}, \tilde{\chi}^0$ .

Impossible in the decoupling limit where these particles are integrated out!

# Summary of large- $\tan\beta$ effects

effect	decoupling limit	beyond
modified relation $y_{d_i} \leftrightarrow m_{d_i}$	[Hall,Rattazzi,Sarid; Carena,Olechowski, Pokorski,Wagner]	[Carena,Garcia, Nierste,Wagner], <span style="border: 1px solid blue; padding: 2px;">1</span>
corrections to CKM matrix	[Blazek,Raby,Pokorski]	[Buras,Chankowski, Rosiek,Slawianowska], <span style="border: 1px solid blue; padding: 2px;">2</span>
enhanced FCNCs $d_i d_j H^0/A^0$	[Hamzaoui,Pospelov,Toharia; Babu,Kolda; Buras,Chankowski,Rosiek, Slawianowska]	[Buras,Chankowski, Rosiek,Slawianowska], <span style="border: 1px solid blue; padding: 2px;">3</span>
enhanced FCNCs $d_i \tilde{d}_j \tilde{g}/\tilde{\chi}^0$	<b>not accessible</b>	<span style="border: 1px solid blue; padding: 2px;">3</span>
vertex corrections $\bar{u}_{i,R} d_{j,L} H^+$	[Degrassi,Gambino,Giudice; Carena,Garcia, Nierste,Wagner]	process-dependent (non-universal)

1 – 3 = this talk

Beyond the decoupling limit:

- 1 Scheme dependence of the resummation formula for the Yukawa coupling
- 2 Resummation of flavour-changing self-energies
- 3 New effects in FCNC processes



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## Input schemes for bottom-squark mixing

► Bottom-squark mass matrix:  $\mathcal{M}_b^2 = \begin{pmatrix} m_{b_L}^2 & -y_b^* v_u \mu \\ -y_b v_u \mu^* & m_{b_R}^2 \end{pmatrix}$

► Mixing matrix:  $\tilde{R}_b \mathcal{M}_b^2 \tilde{R}_b^\dagger = \text{diag}(m_{b_1}^2, m_{b_2}^2),$

$$\tilde{R}_b = \begin{pmatrix} \cos \tilde{\theta}_b & \sin \tilde{\theta}_b e^{i\tilde{\phi}_b} \\ -\sin \tilde{\theta}_b e^{-i\tilde{\phi}_b} & \cos \tilde{\theta}_b \end{pmatrix}$$

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- ▶ What to choose as input? → different possibilities, e.g.
- ▶ elements of  $\mathcal{M}_b^2$ :  $m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta$
  - ▶ mass eigenvalues and mixing angle:  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b$
  - ▶ eigenvalues and off-diag. entries of  $\mathcal{M}_b^2$ :  $m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta$

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▶ **Note:**  $\tilde{\theta}_b$  vanishes for  $v/M_{\text{SUSY}} \rightarrow 0$

→ No different input schemes in the decoupling limit.

# Scheme Dependence of the Resummation Formula

- ▶  $m_{\tilde{b}_1}$ ,  $m_{\tilde{b}_2}$  and  $\tilde{\theta}_b$  depend on  $y_b$  through  $\mathcal{M}_{\tilde{b},12}^2 = -y_b^* v_u \mu$ .
- ▶ Change between different input schemes using relations like

$$m_{\tilde{b}_{1,2}}^2 = \frac{1}{2} \left( m_{\tilde{b}_L}^2 + m_{\tilde{b}_R}^2 \pm \sqrt{\left( m_{\tilde{b}_L}^2 - m_{\tilde{b}_R}^2 \right)^2 + 4|y_b v_u \mu|^2} \right),$$

$$\sin 2\tilde{\theta}_b e^{-i\tilde{\phi}_b} = \frac{-2y_b v_u \mu}{m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2}$$

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- ▶  $y_b$ -dependence of  $\Sigma_b^{RL}(y_b) \longleftrightarrow$  choice of input parameters
- ▶  $y_b$ -dependence of  $\Sigma_b^{RL}(y_b)$  determines possible counterterm insertions

**$\Rightarrow$  Resummation formula depends on renormalization scheme!!!**

**Example:**  $\Sigma_{b,\tilde{g}}^{RL} = m_b \Delta_{\tilde{g}}^b = m_b \epsilon_b^{\tilde{g}} \tan \beta$

$$\propto \sin 2\tilde{\theta}_b e^{-i\tilde{\phi}_b} = \frac{-2\mu^* m_b \tan \beta}{m_{b_1}^2 - m_{b_2}^2}$$

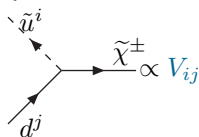
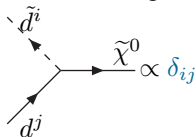
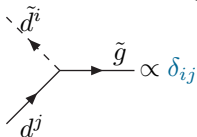
input	$\Sigma_{b,\tilde{g}}^{RL}(y_b) \propto$	resummation formula
$m_{\tilde{b}_1}, m_{\tilde{b}_2}, \tilde{\theta}_b, \tilde{\phi}_b$	$\sin 2\tilde{\theta}_b e^{-i\tilde{\phi}_b}$	$y_b = \frac{m_b}{v_d} \left(1 - \Delta_{\tilde{g}}^b\right)$
$m_{\tilde{b}_1}, m_{\tilde{b}_2}, \mu, \tan \beta$	$\frac{-2y_b v_u \mu}{m_{b_1}^2 - m_{b_2}^2}$	$y_b = \frac{m_b}{v_d(1 + \Delta_{\tilde{g}}^b)}$
$m_{\tilde{b}_L}, m_{\tilde{b}_R}, \mu, \tan \beta$	$\frac{-2y_b v_u \mu}{\sqrt{(m_{b_L}^2 - m_{b_R}^2)^2 + 4 y_b v_u \mu ^2}}$	analytic resummation impossible, use formula (i) iteratively.

Beyond the decoupling limit:

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- ▶ Soft SUSY-breaking terms flavour-diagonal in Super-CKM basis

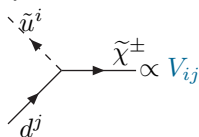
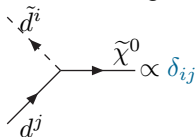
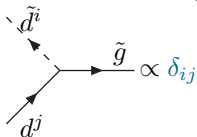


→ Only chargino-loops are flavour-changing

- ▶ Usually this condition is imposed at the **GUT-scale**
  - ▶ RG evolution induces **flavour-violating**  $\tilde{g}$ - and  $\tilde{\chi}^0$ -couplings at the **electro-weak scale** .
  - ▶ Impact of **RG-effects** on FCNC transitions is **small** .

[Baer,Brhlik,Castano,Tata; Dudley,Kolda]

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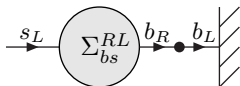


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  - ▶ Impact of **RG-effects** on FCNC transitions is **small** .  
[Baer,Brhlik,Castano,Tata; Dudley,Kolda]
- ▶ We allow for
  - ▶ SUSY-breaking terms of third generation to be different from those of the first two.
  - ▶ flavour-diagonal CP-violating phases (e.g. **complex  $A_t$** )

# Flavour-changing self-energies in external legs

- ▶ Consider flavour-changing self-energies in external quark-legs:



- ▶ New source of  $\tan \beta$ -enhancement:

$$\Sigma_{bs}^{RL} \propto \epsilon_{FC} m_b \tan \beta \quad \text{and} \quad \mathcal{M} \propto \frac{\Sigma_{bs}^{RL}}{m_b} \propto \epsilon_{FC} \tan \beta$$

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- ▶ Subtract self-energies by non-diagonal wave-function CTs:

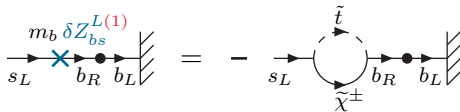
$$\delta Z_{bi}^L \propto \epsilon_{FC} \tan \beta, \quad \delta Z_{bi}^R \propto \frac{m_i}{m_b} \epsilon_{FC} \tan \beta \quad (i = d, s)$$

→  $\delta Z^{L/R}$  contain the  $\tan \beta$ -enhanced effects!

# Resummation of $\delta Z_{ij}^{L,R}$

- ▶ Subtract self-energies by non-diagonal wave-function CTs:

1 loop



$$\delta Z_{bs}^{L(1)} \propto \text{loop} \cdot \tan \beta$$

# Resummation of $\delta Z_{ij}^{L,R}$

- ▶ Subtract self-energies by non-diagonal wave-function CTs:

2 loops

The diagram shows an equality between two Feynman diagrams. On the left is a tree-level diagram representing a 2-loop self-energy correction. It consists of an incoming line labeled  $s_L$  that splits into two lines:  $m_b$  and  $\delta Z_{bs}^{L(2)}$ . These lines then meet at a vertex labeled  $b_R$ , from which a line labeled  $b_L$  goes to a wall representing a boundary. On the right is a diagram representing a 1-loop self-energy correction. It starts with an incoming line  $s_L$  that splits into  $\delta Z_{bs}^{L(1)}$  and a loop. The loop is formed by a solid line labeled  $\tilde{g}$  and a dashed line labeled  $\tilde{b}$ . After the loop, the lines meet at a vertex labeled  $b_R$ , from which a line labeled  $b_L$  goes to a wall. The equation is followed by a minus sign and an ellipsis, indicating that higher-order terms are subtracted.

$$\delta Z_{bs}^{L(2)} \propto (\text{loop} \cdot \tan \beta)^2$$

# Resummation of $\delta Z_{ij}^{L,R}$

- ▶ Subtract self-energies by non-diagonal wave-function CTs:

n loops

$$m_b \delta Z_{bs}^{L(n)} = - \delta Z_{bs}^{L(n-1)} - \dots$$

$$\delta Z_{bs}^{L(n)} \propto (\text{loop} \cdot \tan \beta)^n$$

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$n$  loops

$$m_b \delta Z_{bs}^{L(n)} = - \delta Z_{bs}^{L(n-1)} \tilde{b} - \dots$$

$$\delta Z_{bs}^{L(n)} \propto (\text{loop} \cdot \tan \beta)^n$$

- ▶  $(\text{loop} \cdot \tan \beta)^n$ -effects can be analytically resummed.

- ▶ Results:

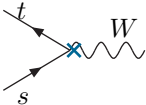
$$\frac{\delta Z_{bi}^L}{2} = -V_{tb}^* V_{ti} \frac{\epsilon_{FC} \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta},$$

$$\frac{\delta Z_{bi}^R}{2} = -V_{tb}^* V_{ti} \frac{m_{d_i}}{m_b} \left[ \frac{\epsilon_{FC} \tan \beta}{1 + \epsilon_b \tan \beta} + \frac{\epsilon_{FC}^* \tan \beta}{(1 + \epsilon_i^* \tan \beta)} \right] \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}$$



# Corrections to the CKM matrix

- ▶  $\delta Z_{ij}^L$  induces corrections of order  $\epsilon_{FC} \tan \beta$  to the  $W$ -vertex:

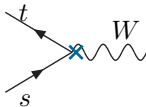


A Feynman diagram showing a vertex where a top quark ( $t$ ) and a strange quark ( $s$ ) meet. The top quark line is incoming from the top-left, and the strange quark line is incoming from the bottom-left. A W boson line, represented by a wavy line, is outgoing to the right. A blue 'x' is placed at the vertex, indicating a correction to the vertex.

$$\propto V_{tb} \frac{\delta Z_{bs}^L}{2} \quad \propto V_{ts} (\epsilon_{FC} \tan \beta)$$

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- ▶ results in corrections to the CKM matrix:

$$V^0 = \begin{pmatrix} V_{ud} & V_{us} & K^* V_{ub} \\ V_{cd} & V_{cs} & K^* V_{cb} \\ KV_{td} & KV_{ts} & V_{tb} \end{pmatrix}, \quad K = \frac{1 + \epsilon_b \tan \beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan \beta}$$

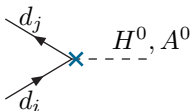
- ▶ This result
  - ▶ is of the **same form** as in the decoupling limit but with **different**  $\epsilon_b, \epsilon_{FC}$ .
  - ▶ is the **analytic expressions** for the limit to which the **iterative calculation** of BCRS converges.

Beyond the decoupling limit:

- 1 Scheme dependence of the resummation formula for the Yukawa coupling
- 2 Resummation of flavour-changing self-energies
- 3 New effects in FCNC processes

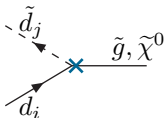
# FCNC-couplings at large $\tan\beta$

- ▶  $\delta Z_{ij}^L$  induce FCNC-couplings of order  $\epsilon_{FC} \tan\beta$ :



known in the decoupling limit

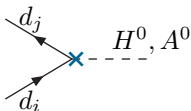
**new:** generalized to  $M_{\text{SUSY}} \sim v$



**new!** (not accessible in the decoupling limit)

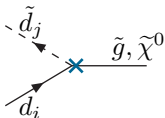
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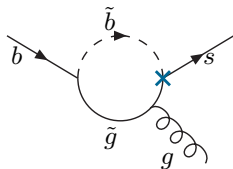
- ▶  $\delta Z_{bi}^L \propto \kappa V_{tb}^* V_{ti} \Rightarrow$  CKM structure of MFV preserved

- ▶ Coupling strength  $\kappa \propto \frac{\epsilon_{FC} \tan\beta}{1 + (\epsilon_b - \epsilon_{FC}) \tan\beta}$

Estimate for equal SUSY-Masses:  $|\kappa| \sim 0.08$ , for  $\mu > 0$   
(larger values for large  $A_t$ )  $|\kappa| \sim 0.24$ , for  $\mu < 0$

## Sizable effect in $C_8$

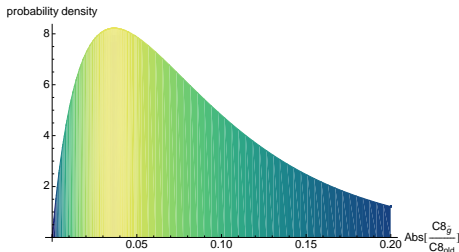
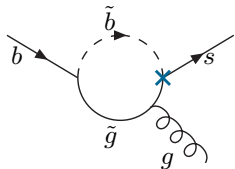
- ▶ Flavour-changing gluino-coupling enters  $\mathcal{H}_{\text{eff}}^{\Delta B=1}$  :
  - ▶ **small effects** in Wilson coefficients of **four-quark operators** and  $C_7$ .
  - ▶ **large effect** in  $C_8$  possible



# Sizable effect in $C_8$

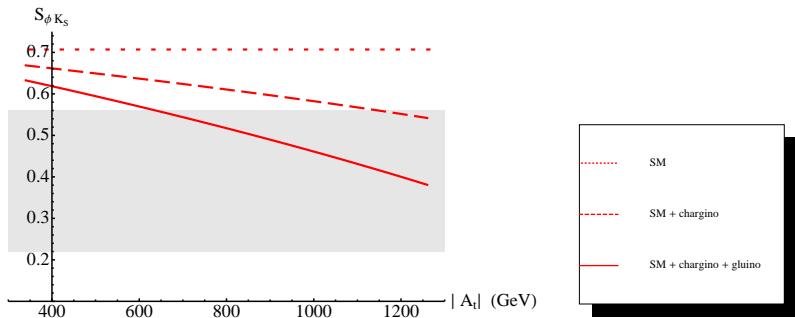
- ▶ Flavour-changing gluino-coupling enters  $\mathcal{H}_{\text{eff}}^{\Delta B=1}$  :
  - ▶ **small effects** in Wilson coefficients of **four-quark operators** and  $C_7$ .
  - ▶ **large effect** in  $C_8$  possible
- ▶ Estimate for equal SUSY-masses:

$$|C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 0.42, \text{ for } \mu > 0; \quad |C_8^{\tilde{g}}/C_8^{\tilde{\chi}^\pm}| \sim 1.3, \text{ for } \mu < 0$$



# Mixing-induced CP asymmetry in $B^0 \rightarrow \phi K_S$

$S_{\phi K_S}$  in naive factorization,  
including  $\tan\beta$ -enhanced corrections to  $C_8$ :

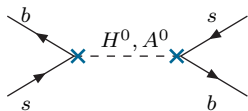


Here a rather large value  $\mu = 800$  GeV is used,  
parameter point is compatible with  $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ .



## ► Neutral Higgs exchange:

- Superficially leading  $C_1^{SLL}$  vanishes because  $H^0$ - and  $A^0$  cancel each other. [Hamzaoui,Pospelov,Toharia]
- $C_2^{LR}$  important despite  $m_s/m_b$  suppression. [Buras,Chankowski,Rosiek,Slawianowska]



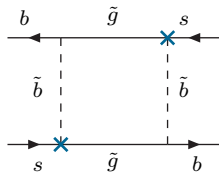
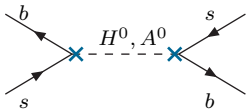
# Contributions to $\mathcal{H}_{eff}^{\Delta B=2}$

## ► Neutral Higgs exchange:

► Superficially leading  $C_1^{SLL}$  vanishes because  $H^0$ - and  $A^0$  cancel each other. [Hamzaoui, Pospelov, Toharia]

►  $C_2^{LR}$  important despite  $m_s/m_b$  suppression. [Buras, Chankowski, Rosiek, Slawianowska]

## ► Gluino-box contributions **small** (GIM suppression)



# CP-violating phase in $C_2^{LR}$

▶ Higgs contribution in [Gorbahn,Jäger,Nierste,Trine]:  $\tilde{C}_2^{LR} = \text{real}$

▶ We find for the Higgs contribution to  $C_2^{LR}$  in our scenario:

$$C_2^{LR} = \tilde{C}_2^{LR}(1 + r), \quad \text{with} \quad r = (1 - e^{2i\phi}) \frac{(\epsilon_b^* - \epsilon_{FC}^* - \epsilon_s^*) \tan \beta}{1 + \epsilon_s^* \tan \beta},$$

$$\phi = \arg \{ \epsilon_{FC} \tan \beta (1 + (\epsilon_b^* - \epsilon_{FC}^*) \tan \beta) \}$$

▶ The correction term  $r$  disappears if

- ▶ all parameters are real or
- ▶ all squark masses are equal.

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- ▶ The correction term  $r$  disappears if
  - ▶ all parameters are real or
  - ▶ all squark masses are equal.
- ▶ Beyond the decoupling limit squark masses are split due to electro-weak symmetry breaking  $\rightarrow$  small effect:

$$|r| \lesssim 0.01, \text{ for } \mu > 0, \quad |r| \lesssim 0.1, \text{ for } \mu < 0.$$

- ▶ Larger effects possible for non-universal squark mass terms

# Conclusions

- ▶ Effects of  $\tan\beta$ -enhanced self-energies can be **resummed analytically beyond the decoupling limit**, also in the flavour-non-diagonal case.
- ▶ The **resummation formula** for the Yukawa coupling depends on the **renormalization scheme**.
- ▶ Not only  $H^0, A^0$  but also  $\tilde{g}, \tilde{\chi}^0$  develop **flavour-changing couplings** at large  $\tan\beta$ .
- ▶ These couplings lead to a **sizable modification of  $C_8$** .

# Backup slides

## Backup: Parameter points

Scan ranges for  $C_8$ :  $\tan \beta = 40 - 60$ , any value for  $\varphi_{A_t}$ ,

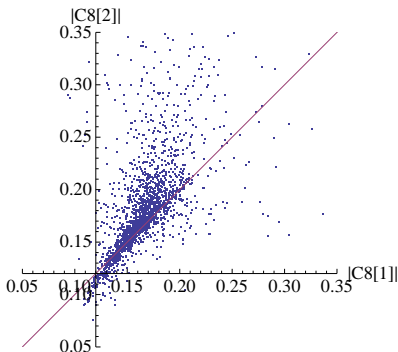
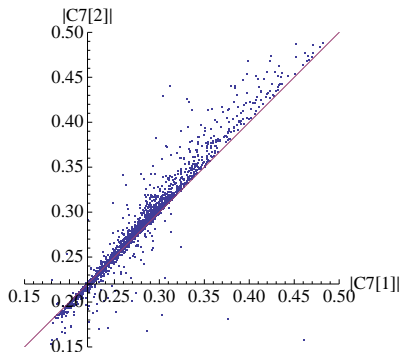
	min (GeV)	max (GeV)
$\tilde{m}_{Q_L}, \tilde{m}_{u_R}, \tilde{m}_{d_R}$	200	1000
$ A_t $	100	1000
$\mu, M_1, M_2$	200	1000
$M_3$	300	1000
$m_{H^+}$	200	1000

Parameter point used for  $S_{\phi_{K_S}}$ :

$\tilde{m}_{Q_L}, \tilde{m}_{u_R}, \tilde{m}_{d_R}$	600 GeV	$\tan \beta$	50
$\mu$	800 GeV	$m_{A^0}$	350 GeV
$M_1$	300 GeV	$M_2$	400 GeV
$M_3$	500 GeV	$\varphi_{A_t}$	$3\pi/2$

## Backup: The Wilson coefficients $C_7$ and $C_8$

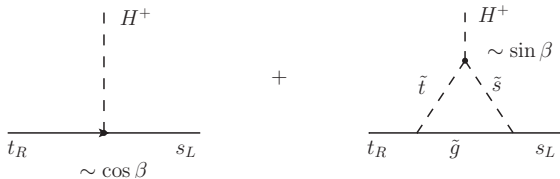
- ▶  $C_{7,8}[1] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+}$ ,     $C_{7,8}[2] = C_{7,8}^{\tilde{\chi}^\pm} + C_{7,8}^{H^+} + C_{7,8}^{\tilde{g}}$
- ▶ Scan over relevant SUSY parameter space with  
 $(\mu, M_1, M_2, m_{\tilde{g}}, M_{H^+}, m_{\tilde{t},LL}, m_{\tilde{t},RR}, m_{\tilde{b},LL}, m_{\tilde{b},RR}) \leq 1\text{TeV}$ ,  
 $|A_t| \leq 3\text{TeV}$ ,     $0 \leq \phi_{A_t} \leq 2\pi$ ,     $\tan\beta = 50$





## Backup: Non-local $\tan\beta$ -enhanced effects

- ▶ Some couplings of  $H^+$  and  $h^0$  are suppressed by  $\cos\beta$  at tree-level.
- ▶ They obtain enhanced vertex corrections  $\sim \sin\beta$ , e.g.



- ▶ This effect is local only in the decoupling limit, but cannot be cast into a Feynman rule in the full calculation.