Three-loop fermion and gluon form factors

Matthias Steinhauser

TTP, University of Karlsruhe

Flavianet Workshop, July 23-27 2009, Kazimierz

Universität Karlsruhe (TH)

Forschungsuniversität • gegründet 1825

Karlsruhe Institute of Technology

Outline

I. Massless quark and gluon form factor

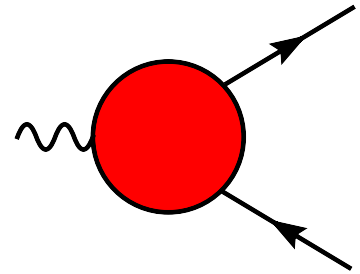
with P.A. Baikov, K.G. Chetyrkin, A.V Smirnov, V.A. Smirnov

II. $gg \to H$ @ NNLO beyond $m_t \to \infty$: virtual corrections

with A. Pak, M. Rogal

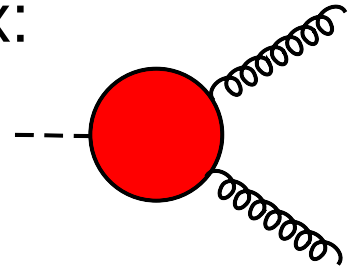
I. Form factor

Photon-quark vertex:



$\Rightarrow \quad \gamma^\mu F_q(q^2)$

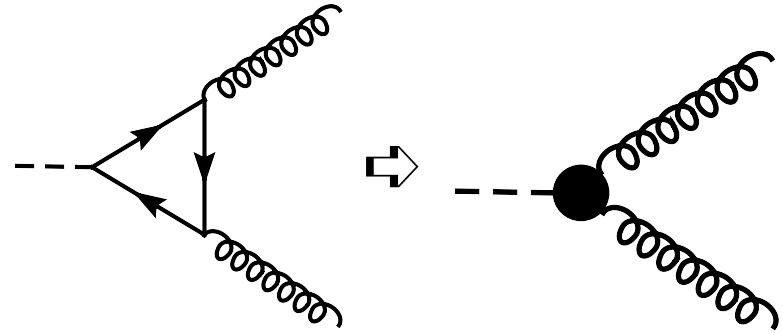
Higgs-gluon vertex:



$\Rightarrow \quad \left(g^{\mu\nu} q_1 q_2 - q_1^\nu q_2^\mu\right) F_g(q^2)$

F_g: construct effective theory for $m_t \to \infty$:

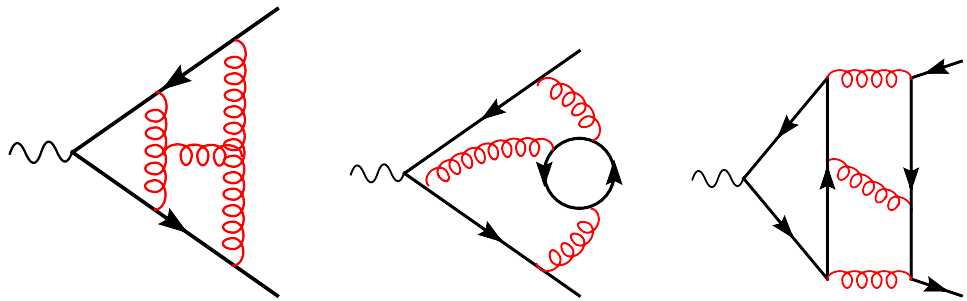
$\mathcal{L}^{\rm SM} \Rightarrow \mathcal{L}_{\rm eff} = C_1 \frac{H}{v} G_{\mu\nu} G^{\mu\nu} + \ldots$



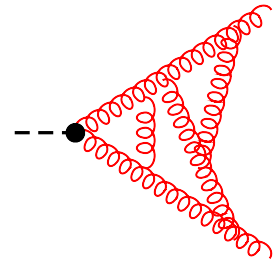
Form factor (2)

$$F = 1 + \frac{\alpha_s}{\pi} F^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 F^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 F^{(3)}$$

$$F_q^{(3)} = F_q^{(3),g} + F_q^{(3),n_f} + \sum_{q'} Q_{q'} F_q^{(3),sing}$$



$F_g^{(3)}$



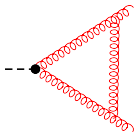
Known results

- $F_q^{(2)}$ [(Gonsalves'83); Kramer,Lampe'86; Matsuura, van Neerven'87; Matsuura, van der Marck, van Neerven'88]
- $F_g^{(2)}$ [Harlander'00; Ravindran,Smith,van Neerven'04]
- $F_q^{(2)}$ and $F_g^{(2)}$ to $\mathcal{O}(\epsilon^2)$ [Gehrmann,Huber,Maitre'05]

Known results

- $F_q^{(2)}$ [(Gonsalves'83); Kramer,Lampe'86; Matsuura, van Neerven'87; Matsuura, van der Marck, van Neerven'88]
- $F_g^{(2)}$ [Harlander'00; Ravindran,Smith,van Neerven'04]
- $F_q^{(2)}$ and $F_g^{(2)}$ to $\mathcal{O}(\epsilon^2)$ [Gehrmann,Huber,Maitre'05]

$$F_g^{(1)} = C_A \left\{ -\frac{2}{\epsilon^2} + \zeta_2 + \epsilon \left(-2 + \frac{14}{3}\zeta_3 \right) + \epsilon^2 \left(-6 + \frac{47}{20}\zeta_2^2 \right) + \epsilon^3 \left(-14 + \zeta_2 - \frac{7}{3}\zeta_2\zeta_3 + \frac{62}{5}\zeta_5 \right) + \epsilon^4 \left(-30 + 3\zeta_2 + \frac{14}{3}\zeta_3 + \frac{949}{280}\zeta_2^3 - \frac{49}{9}\zeta_3^2 \right) \right\}$$



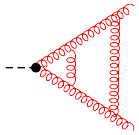
Known results

$$
\begin{aligned}
F_g^{(2)} \quad = \quad & C_A^2 \Bigg\{ \frac{2}{\epsilon^4} - \frac{11}{6\epsilon^3} + \frac{1}{\epsilon^2}\left(-\frac{67}{18} - \zeta_2\right) + \frac{1}{\epsilon}\left(\frac{68}{27} + \frac{11}{2}\zeta_2 - \frac{25}{3}\zeta_3\right) + \frac{5861}{162} + \frac{67}{6}\zeta_2 \\
& + \frac{11}{9}\zeta_3 - \frac{21}{5}\zeta_2^2 + \epsilon\left(\frac{158201}{972} + \frac{106}{9}\zeta_2 - \frac{1139}{27}\zeta_3 - \frac{77}{60}\zeta_2^2 + \frac{23}{3}\zeta_2\zeta_3 + \frac{71}{5}\zeta_5\right) \\
& + \epsilon^2\left(\frac{3484193}{5832} + \frac{481}{54}\zeta_2 - \frac{26218}{81}\zeta_3 - \frac{1943}{60}\zeta_2^2 - \frac{55}{3}\zeta_2\zeta_3 + \frac{341}{15}\zeta_5 + \frac{2313}{70}\zeta_2^3\right. \\
& \left. + \frac{901}{9}\zeta_3^2\right)\Bigg\} + C_A n_f \Bigg\{ \frac{1}{3\epsilon^3} + \frac{5}{9\epsilon^2} + \frac{1}{\epsilon}\left(-\frac{26}{27} - \zeta_2\right) - \frac{808}{81} - \frac{5}{3}\zeta_2 - \frac{74}{9}\zeta_3 \\
& + \epsilon\left(-\frac{23131}{486} - \frac{16}{9}\zeta_2 - \frac{604}{27}\zeta_3 - \frac{51}{10}\zeta_2^2\right) + \epsilon^2\left(-\frac{540805}{2916} + \frac{28}{27}\zeta_2 - \frac{3962}{81}\zeta_3\right. \\
& \left. - \frac{257}{18}\zeta_2^2 + \frac{50}{3}\zeta_2\zeta_3 - \frac{542}{15}\zeta_5\right)\Bigg\} + C_F n_f \Bigg\{ -\frac{1}{\epsilon} - \frac{67}{6} + 8\zeta_3 + \epsilon\left(-\frac{2027}{36}\right. \\
& \left. + \frac{7}{3}\zeta_2 + \frac{92}{3}\zeta_3 + \frac{16}{3}\zeta_2^2\right) + \epsilon^2\left(-\frac{47491}{216} + \frac{209}{18}\zeta_2 + \frac{1124}{9}\zeta_3 + \frac{184}{9}\zeta_2^2\right. \\
& \left. - \frac{40}{3}\zeta_2\zeta_3 + 32\zeta_5\right)\Bigg\}
\end{aligned}
$$

88]

04]

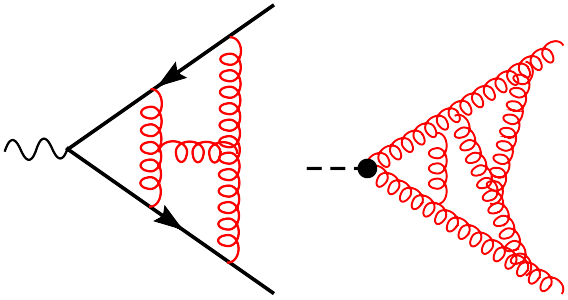
5]



Known results

- $F_q^{(2)}$ [(Gonsalves'83); Kramer,Lampe'86; Matsuura, van Neerven'87; Matsuura, van der Marck, van Neerven'88]
- $F_g^{(2)}$ [Harlander'00; Ravindran,Smith,van Neerven'04]
- $F_q^{(2)}$ and $F_g^{(2)}$ to $\mathcal{O}(\epsilon^2)$ [Gehrmann,Huber,Maitre'05]
- $1/\epsilon$ poles for $F_q^{(3)}$ and $F_g^{(3)}$ [Moch,Vermaseren,Vogt'05]

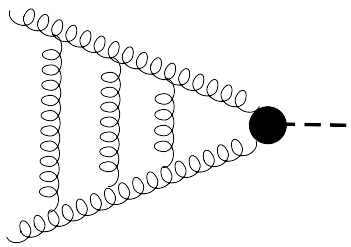
$$F^{(3)} = \frac{\#}{\epsilon^6} + \frac{\#}{\epsilon^5} + \frac{\#}{\epsilon^4} + \frac{\#}{\epsilon^3} + \frac{\#}{\epsilon^2} + \frac{\#}{\epsilon^1} + \boxed{??}$$



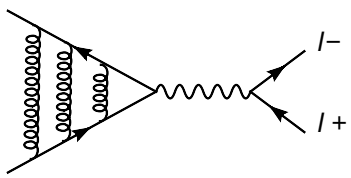
Applications

Virtual NNNLO corrections to

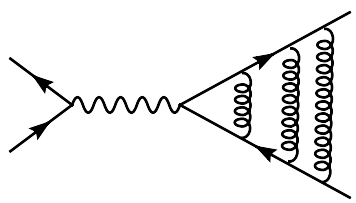
- $gg \to H$



- DY

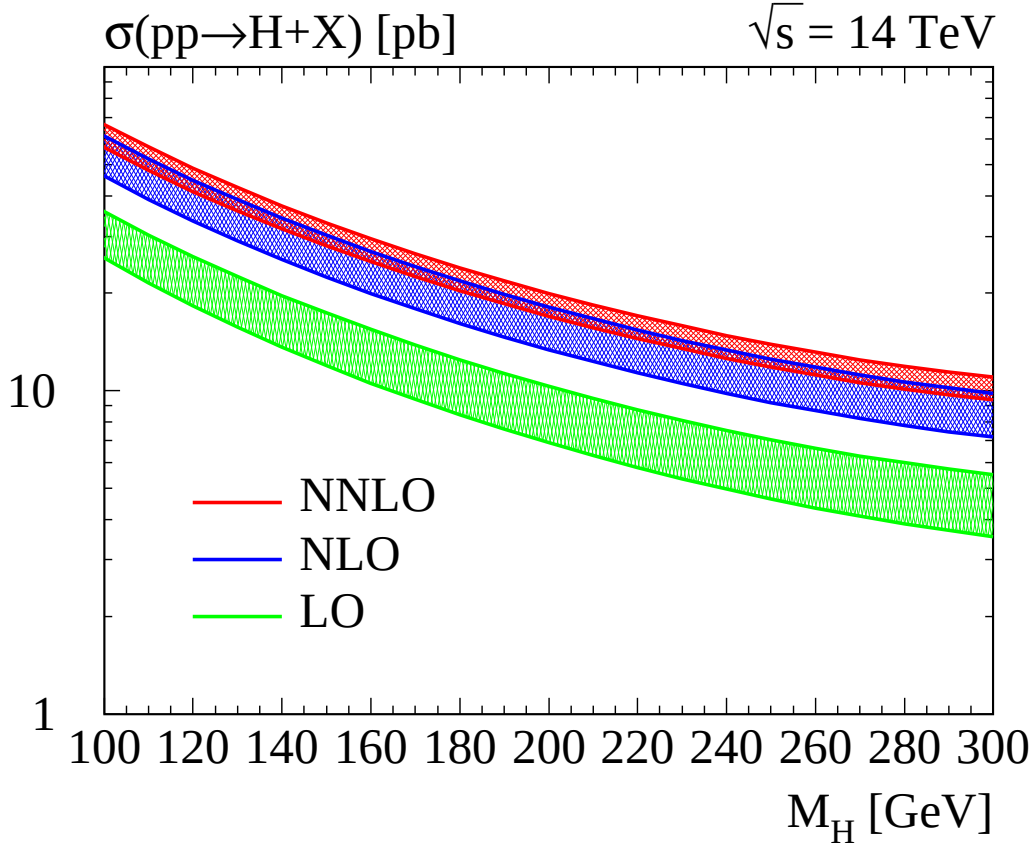


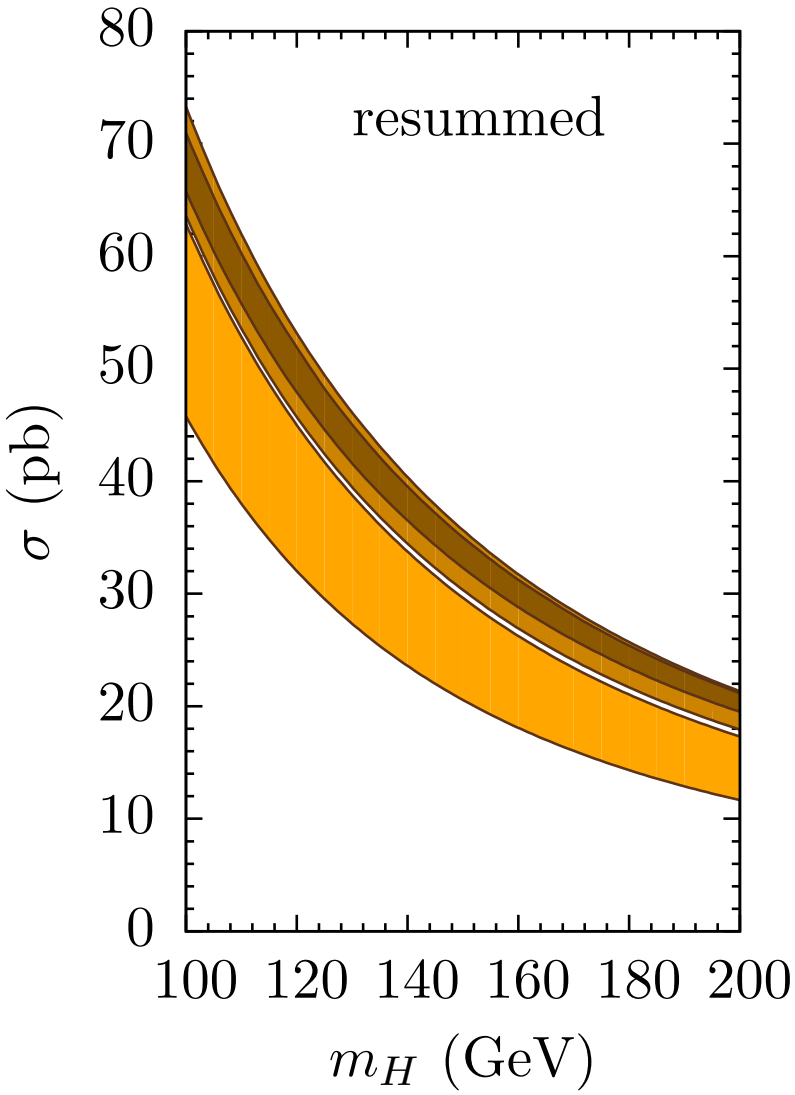
- $e^+e^- \to 2$ Jets



- soft-gluon resummation

Applications





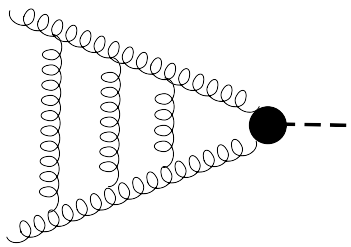
[Harlander,Kilgore'02], [Anastasiou,Melnikov'02], [Ravindran,Smith,v.Neerven'03]

[Catani,de Florian,Grazzini,Nason'03] [Ahrens,Becher,Neubert,Yang'08]

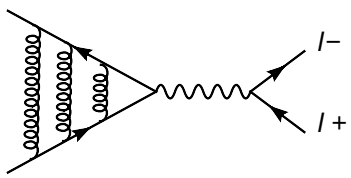
Applications

Virtual NNNLO corrections to

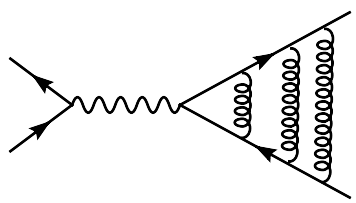
- $gg \to H$



- DY



- $e^+e^- \to 2$ Jets



- soft-gluon resummation

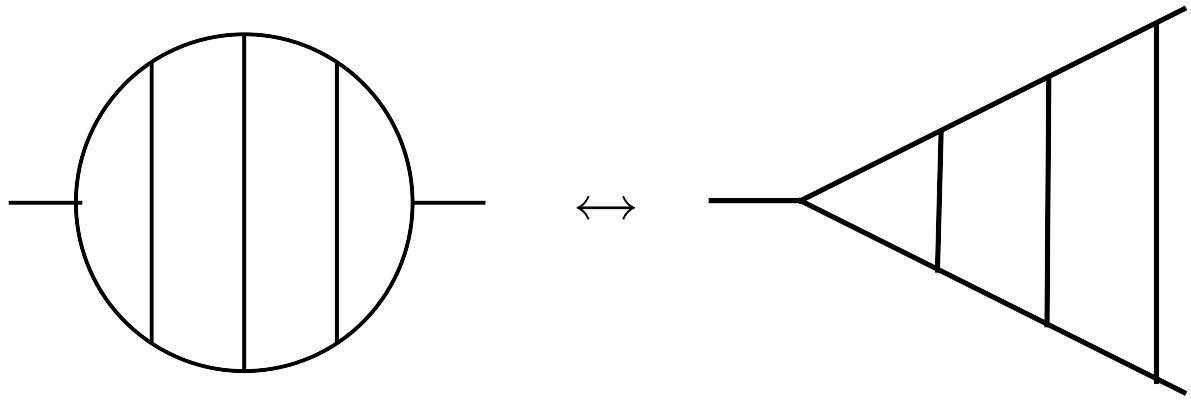
Some technical details

1. Reduction to MIs
2. Compute MIs

1. Reduction to MIs

Main approach:

- equivalence for recurrence relations between N-loop 2-point and $(N-1)$-loop 3-point functions [Baikov,Smirnov'96]



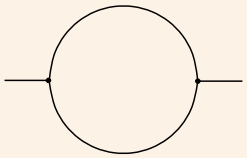
- similar to `BAICER` [Baikov,Chetyrkin,Kühn'02...'08]
- `ParFORM`, `TFORM` [Tentyukov,Vermaseren,...'04...'09]
- "Baikov-method"

1. Reduction to MIs

- Baikov's method: $I = \sum_{MI}$ "coef" $\times\, MI$ [Baikov'96,...,Smirnov,MS'03]
- integral representation for "coef"

$$\sim \int \ldots \int \frac{\mathrm{d}x_1 \ldots \mathrm{d}x_N}{x_1^{n_1} \ldots x_N^{n_N}} \left[P(x_1, \ldots)\right]^{(d-h-1)/2}$$

- (simple) example:



$$F(n_1, n_2) = \int \frac{\mathrm{d}^d k}{(k^2)^{n_1} [(k-q)^2]^{n_2}} = c_1(n_1, n_2)\, F(1,1)$$

$$P(x_1, x_2) = (q^2)^2 - 2q^2(x_1 + x_2) + (x_1 - x_2)^2$$

$$c_1(n_1, n_2) = \frac{(q^2)^{(d-3)}}{(n_1-1)!} \left(\frac{\partial}{\partial x_1}\right)^{n_1-1} \frac{1}{(n_2-1)!} \left(\frac{\partial}{\partial x_2}\right)^{n_2-1} \left[P(x_1, x_2)\right]^{(d-3)/2} \Bigg|_{x_i=0}$$

- in general: compute for $d \to \infty$ ➪ reconstruct "coef" [Baikov,Chetyrkin,Kühn'02...'08]

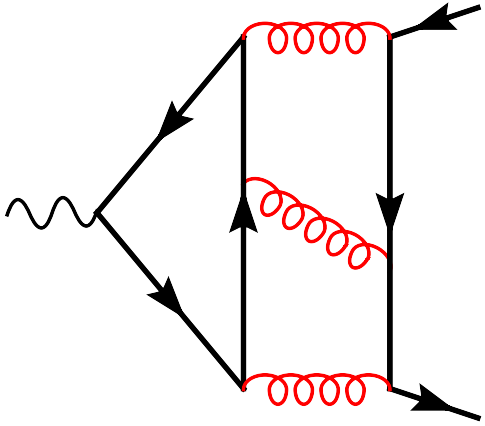
1. Reduction to MIs

Main approach:

- equivalence for recurrence relations between N-loop 2-point and $(N-1)$-loop 3-point functions [Baikov,Smirnov'96]
- similar to `BAICER` [Baikov,Chetyrkin,Kühn'02...'08]
- `ParFORM`, `TFORM` [Tentyukov,Vermaseren,...'04...'09]
- "Baikov-method"

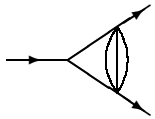
For singlet diagrams:

independent calculation with `FIRE`: Laporta $\oplus$ Gröbner [Smirnov'08]

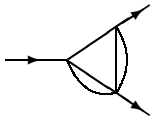


➪ 22 MIs

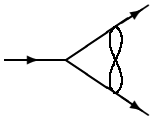
2. MIs



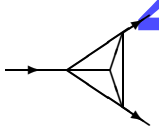
$A_{5,1}$



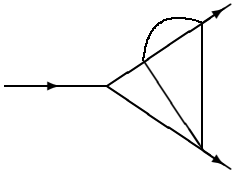
$A_{5,2}$



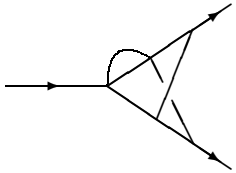
$A_{6,1}$



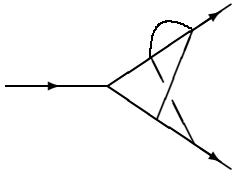
$A_{6,2}$



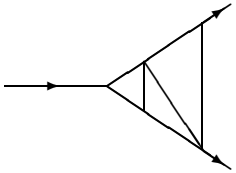
$A_{6,3}$



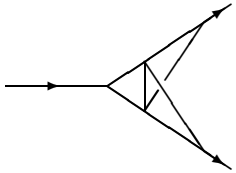
$A_{7,1}$



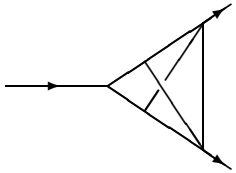
$A_{7,2}$



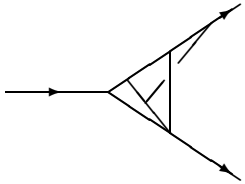
$A_{7,3}$



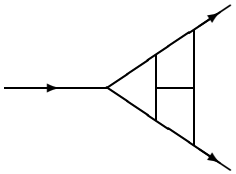
$A_{7,4}$



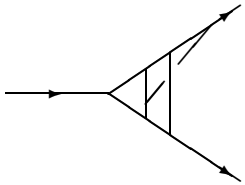
$A_{7,5}$



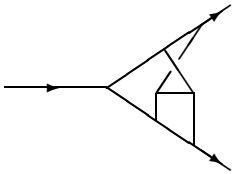
A_8



$A_{9,1}$



$A_{9,2}$



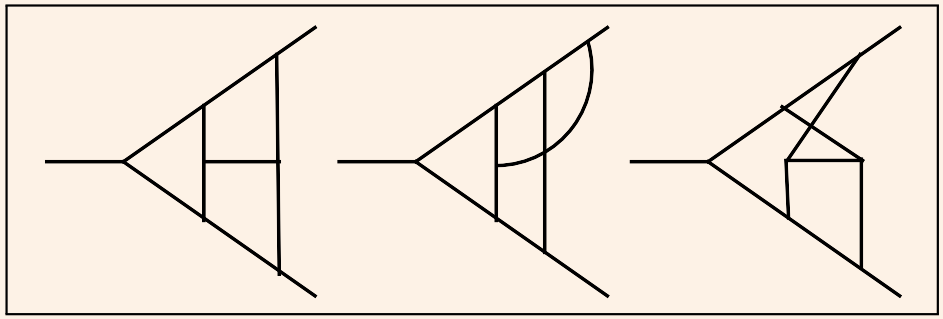
$A_{9,4}$

(+ 8 simple MIs)

2 MIs

22 master integrals

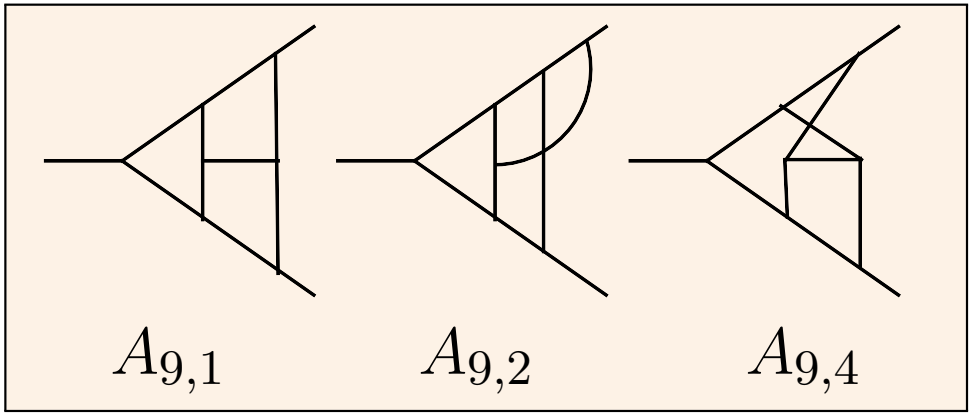
- 19 MIs: [Gehrmann,Heinrich,Huber,Studerus'06; Heinrich,Huber,Maitre'08]
- 3 most complicated MIs:



- extract by comparison with $F_q\Big|_{1/\epsilon-\text{poles}}$ [Moch,Vermaseren,Vogt'05]

  $F_g\Big|_{1/\epsilon-\text{poles}}$: consistency check
- 3 remaining coefficients in ϵ expansion: `FIESTA` [Smirnov,Tentukov'08], Mellin Barnes, `MB` [Czakon'06]
- independent (explicit) calculation: [Heinrich,Huber,Kosower,Smirnov'09]

2. MIs (2)



[Baikov,Chetyrkin,Smirnov,Smirnov,MS'09]

[Heinrich,Huber,Kosower,Smirnov'09]

$$
\begin{aligned}
A_{9,4} \quad = \quad & -\frac{1}{9\epsilon^6} - \frac{8}{9\epsilon^5} + \frac{1}{\epsilon^4}\left(1 + \frac{43\zeta(2)}{18}\right) + \frac{1}{\epsilon^3}\left(\frac{14}{9} + \frac{106\zeta(2)}{9} + \frac{109\zeta(3)}{9}\right) \\
& + \frac{1}{\epsilon^2}\left(-17 - \frac{311\zeta(2)}{18} + \frac{608\zeta(3)}{9} - \frac{481\zeta(4)}{144}\right) \\
& + \frac{1}{\epsilon}\left(84 + \frac{11\zeta(2)}{3} - \frac{949\zeta(3)}{9} + \frac{425\zeta(4)}{6} + \frac{3463\zeta(5)}{45} - \frac{2975\zeta(2)\zeta(3)}{18}\right) \\
& + X_{9,4} + \mathcal{O}(\epsilon)
\end{aligned}
$$

Results

- $F_q^{(3)}$ and $F_g^{(3)}$: agreement with poles [Moch,Vermaseren,Vogt'05]
- $F_q^{(3)}\Big|_{n_f-}$part [Moch,Vermaseren,Vogt'05]
- $F_q^{(3),sing}$ can be extracted from [Moch,Vermaseren,Vogt'04]

Results

[Baikov,Chetyrkin,Smirnov,Smirnov,MS'09]

$$\boxed{F_q^{(3),g+n_f}\Big|_{\rm fin}} = C_F^3\left(\frac{26871}{8}-\frac{95137\zeta(2)}{60}+\frac{5569\zeta(3)}{5}+\frac{95375\zeta(4)}{48}+\frac{30883\zeta(2)\zeta(3)}{15}-\frac{16642\zeta(5)}{5}+\frac{2669(\zeta(3))^2}{3}\right.$$
$$\left.+\frac{1961387\zeta(6)}{2880}-\frac{24X_{9,1}}{5}+\frac{24X_{9,2}}{5}+\frac{6X_{9,4}}{5}\right)+C_AC_F^2\left(\frac{20003431}{29160}+\frac{4239679\zeta(2)}{1620}-\frac{121753\zeta(3)}{30}\right.$$
$$-\frac{11155817\zeta(4)}{4320}-\frac{92554\zeta(2)\zeta(3)}{45}+\frac{610462\zeta(5)}{225}-\frac{36743(\zeta(3))^2}{30}-\frac{1118529\zeta(6)}{640}+\frac{24X_{9,1}}{5}$$
$$\left.-\frac{16X_{9,2}}{5}-\frac{9X_{9,4}}{5}\right)+C_A^2C_F\left(-\frac{88822328}{32805}-\frac{3486997\zeta(2)}{2916}+\frac{3062512\zeta(3)}{1215}+\frac{4042277\zeta(4)}{4320}\right.$$
$$\left.+\frac{5233\zeta(2)\zeta(3)}{12}-\frac{202279\zeta(5)}{450}+\frac{63043(\zeta(3))^2}{180}+\frac{4741699\zeta(6)}{11520}-X_{9,1}+\frac{2X_{9,2}}{5}+\frac{3X_{9,4}}{5}\right)$$
$$+C_F^2n_fT\left(-\frac{2732173}{1458}-\frac{45235\zeta(2)}{81}+\frac{102010\zeta(3)}{81}+\frac{40745\zeta(4)}{216}-\frac{686\zeta(3)\zeta(2)}{9}+\frac{556\zeta(5)}{45}\right)$$
$$+C_AC_Fn_fT\left(\frac{17120104}{6561}+\frac{442961\zeta(2)}{729}-\frac{90148\zeta(3)}{81}-\frac{5465\zeta(4)}{27}+\frac{736\zeta(3)\zeta(2)}{9}-\frac{416\zeta(5)}{3}\right)$$
$$+C_Fn_f^2T^2\left(-\frac{2710864}{6561}-\frac{248\zeta(2)}{3}+\frac{12784\zeta(3)}{243}-\frac{166\zeta(4)}{27}\right)$$

$$X_{9,1}\approx 1428.9963678666183591\,,$$
$$X_{9,2}\approx 528.0583\pm 0.0326\,,\; X_{9,4}\approx -2085.380547\pm 0.000025$$

Results

[Baikov,Chetyrkin,Smirnov,Smirnov,MS'09]

$$\boxed{F_q^{(3),sing}\Big|_{\rm fin}} = d^{abc}d^{abc}\left(\frac{2}{3}+\frac{5\zeta(2)}{3}+\frac{7\zeta(3)}{9}-\frac{\zeta(4)}{6}-\frac{40\zeta(5)}{9}\right)$$

$$\boxed{F_g^{(3)}\Big|_{\rm fin}} = C_A^3\left(\frac{14423912}{6561}+\frac{384479\zeta(2)}{2916}-\frac{370649\zeta(3)}{486}+\frac{280069\zeta(4)}{864}+\frac{1821\zeta(2)\zeta(3)}{4}-\frac{66421\zeta(5)}{90}\right.$$
$$\left.+\frac{545(\zeta(3))^2}{36}-\frac{167695\zeta(6)}{256}-X_{9,1}+2X_{9,2}\right)+C_A^2 n_f T\left(-\frac{10021313}{6561}-\frac{75736\zeta(2)}{729}-\frac{1508\zeta(3)}{27}\right.$$
$$\left.+\frac{437\zeta(4)}{12}-\frac{878\zeta(3)\zeta(2)}{9}+\frac{6476\zeta(5)}{45}\right)+C_F C_A n_f T\left(-\frac{155629}{243}-\frac{82\zeta(2)}{3}+\frac{23584\zeta(3)}{81}-16\zeta(4)\right.$$
$$\left.+96\zeta(3)\zeta(2)+\frac{64\zeta(5)}{9}\right)+C_F^2 n_f T\left(\frac{608}{9}+\frac{592\zeta(3)}{3}-320\zeta(5)\right)+C_F n_f^2 T^2\left(\frac{42248}{81}-\frac{64\zeta(2)}{3}\right.$$
$$\left.-\frac{2816\zeta(3)}{9}-\frac{224\zeta(4)}{3}\right)+C_A n_f^2 T^2\left(\frac{2958218}{6561}+\frac{304\zeta(2)}{27}+\frac{47296\zeta(3)}{243}+\frac{1594\zeta(4)}{27}\right)$$

$$X_{9,1} \approx 1428.9963678666183591\,,$$
$$X_{9,2} \approx 528.0583 \pm 0.0326\,,\; X_{9,4} \approx -2085.380547 \pm 0.000025$$

Results

$$X_{9,1} \approx 1428.9963678666183591\,,$$

[Baikov,Chetyrkin,Smirnov,Smirnov,MS'09]

$$X_{9,2} \approx 528.0583 \pm 0.0326\,,\; X_{9,4} \approx -2085.380547 \pm 0.000025$$

SYM ($C_A = C_F = 2T$, $n_f = 1$):

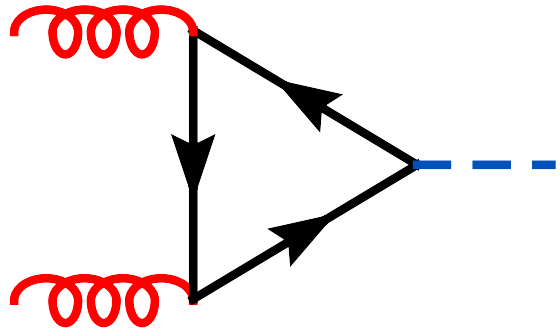
$$F_q^{(3),g+n_f}\Big|_{\rm fin} = C_A^3 \left(\frac{389216}{243} - \frac{155935\zeta(2)}{972} - \frac{54703\zeta(3)}{162} + \frac{23897\zeta(4)}{72} + \frac{15875\zeta(2)\zeta(3)}{36} - \frac{11279\zeta(5)}{10} + \frac{545(\zeta(3))^2}{36} - \frac{167695\zeta(6)}{256} - X_{9,1} + 2X_{9,2} \right)$$

$$F_g^{(3)}\Big|_{\rm fin} = C_A^3 \left(\frac{676219}{486} + \frac{61937\zeta(2)}{972} - \frac{93295\zeta(3)}{162} + \frac{95171\zeta(4)}{288} + \frac{16361\zeta(2)\zeta(3)}{36} - \frac{1645\zeta(5)}{2} + \frac{545(\zeta(3))^2}{36} - \frac{167695\zeta(6)}{256} - X_{9,1} + 2X_{9,2} \right)$$

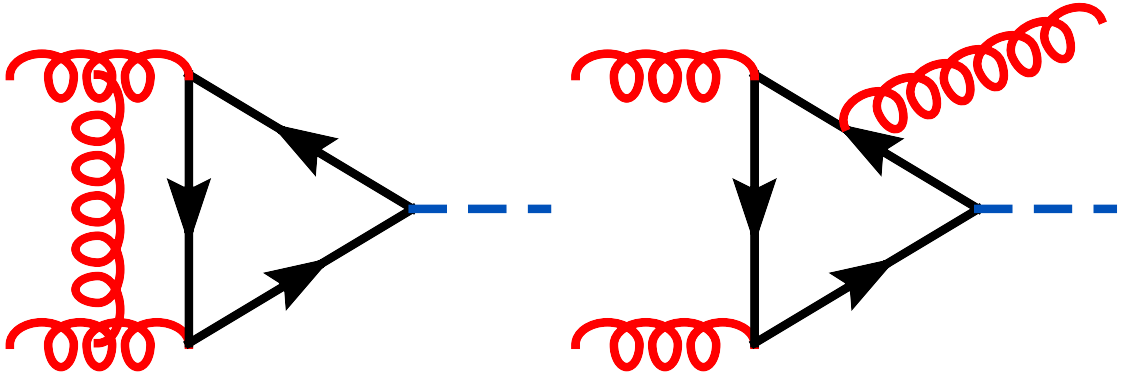
II. $gg \to H$ beyond $m_t \to \infty$

Higgs production in gluon fusion at LHC

LO



NLO

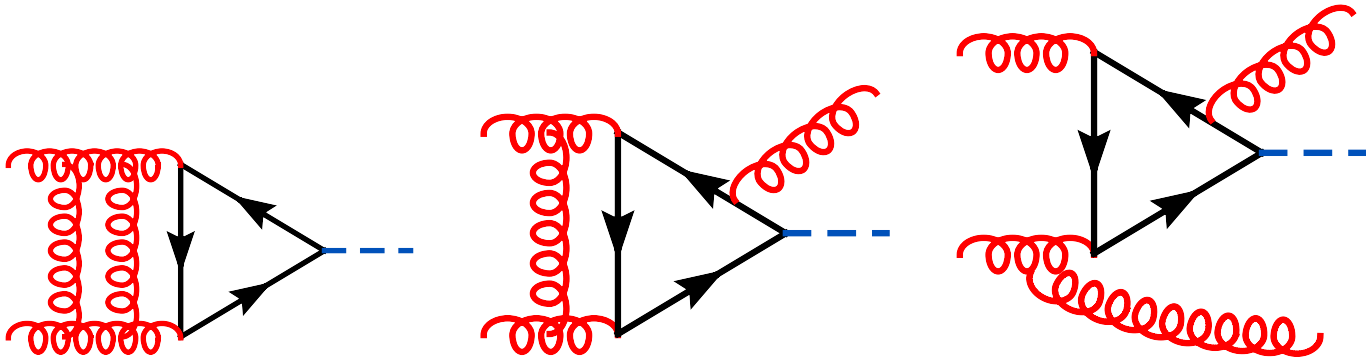


complete dependence on $\hat{s}, m_f, M_H$

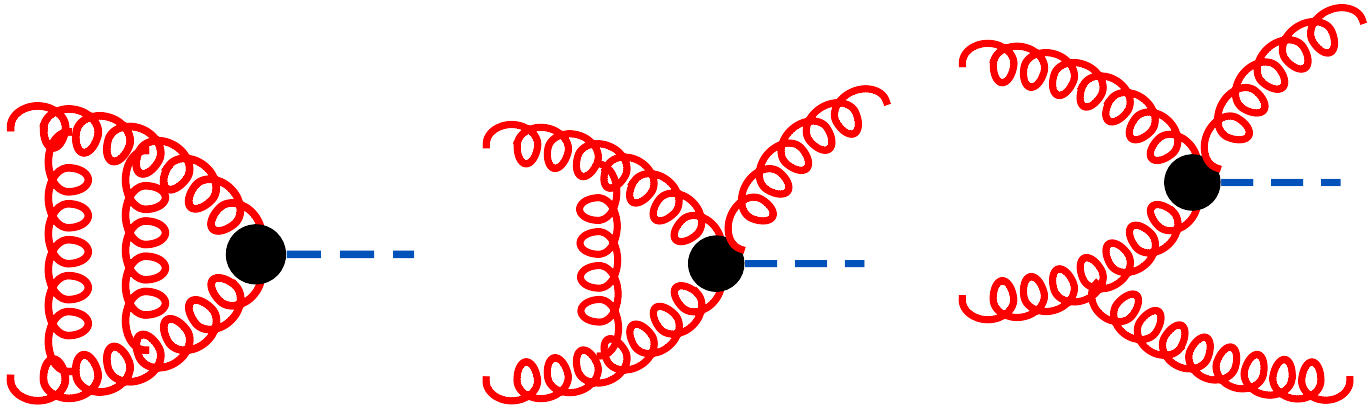
[Dawson'90; Spira,Djouadi,Graudenz,Zerwas'95]

$gg \to H$ @ NNLO

Higgs production in gluon fusion at LHC



$\Downarrow \quad m_t \to \infty: \quad \mathcal{L}^{\text{eff}} = \frac{H}{v} C_1 \left(G^a_{\mu\nu}\right)^2 \quad \Downarrow$

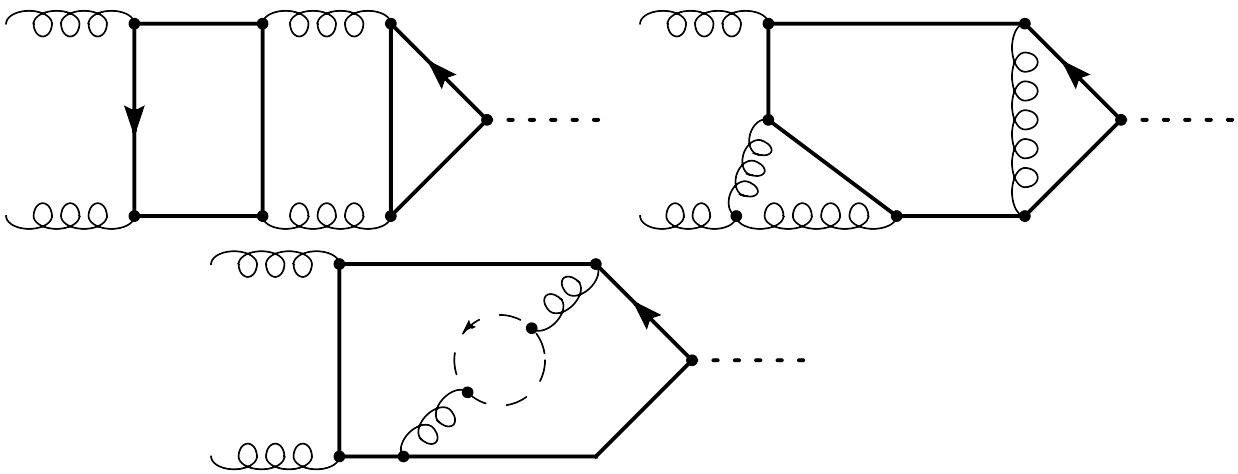


[C_1: Chetyrkin,Kniehl,MS'97]

[σ_{ggH}: Harlander'00; Harlander,Kilgore'02; Anastasiou,Melnikov'02; Ravindran,Smith,van Neerven'03]

$1/m_t$ corrections

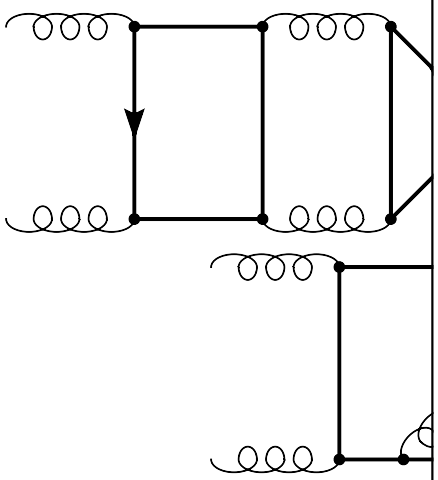
- NLO: “full” vs. “$m_t \to \infty$” $\longrightarrow$ few %
- NNLO: $1/m_t$ corrections ?
- 1. step: virtual corrections

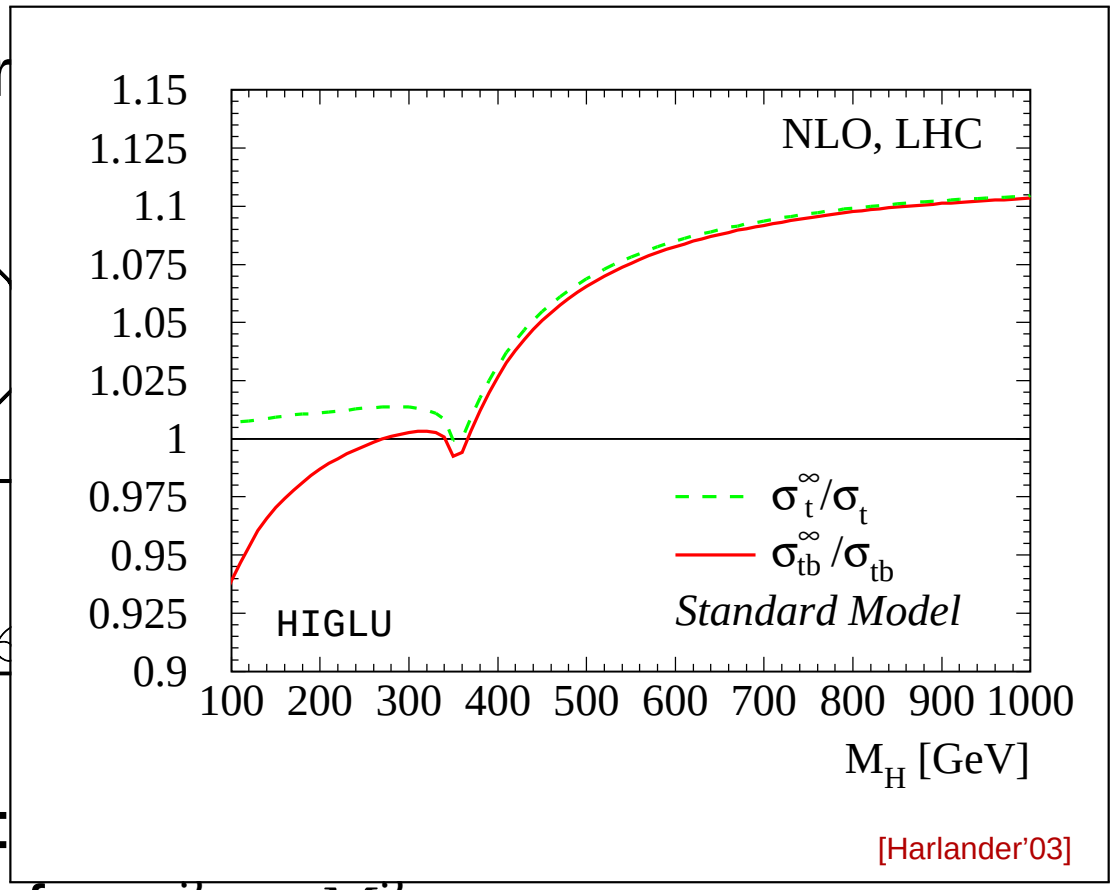


- promising:
  expansion for $m_t^2 \gg M_H^2$

$1/m_t$ corrections

- NLO: “full” vs. “$m_t \to \infty$” $\longrightarrow$ few %
- NNLO: $1/m_t$ corrections ?
- 1. step: vir

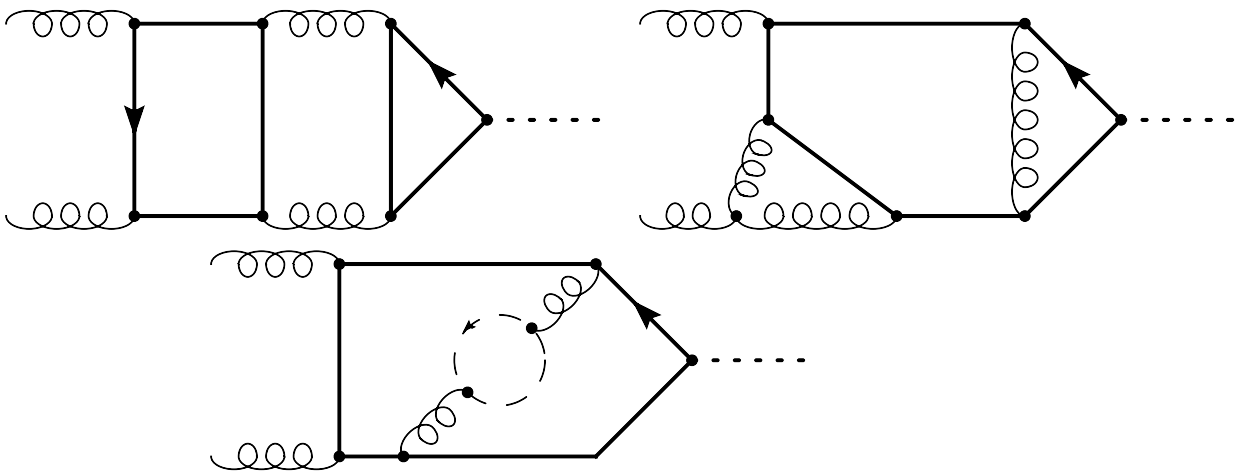




- promising:
  expansion for $m_t^2 \gg M_H^2$

$1/m_t$ corrections

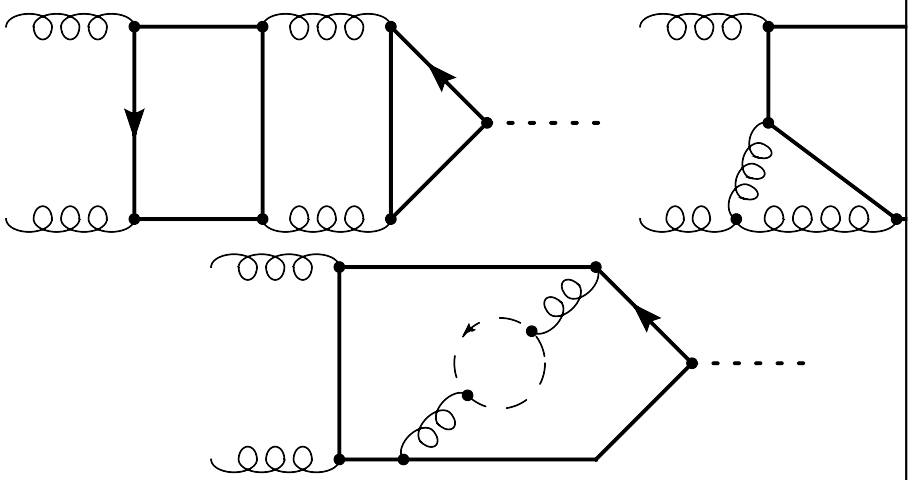
- NLO: “full” vs. “$m_t \to \infty$” $\longrightarrow$ few %
- NNLO: $1/m_t$ corrections ?
- 1. step: virtual corrections

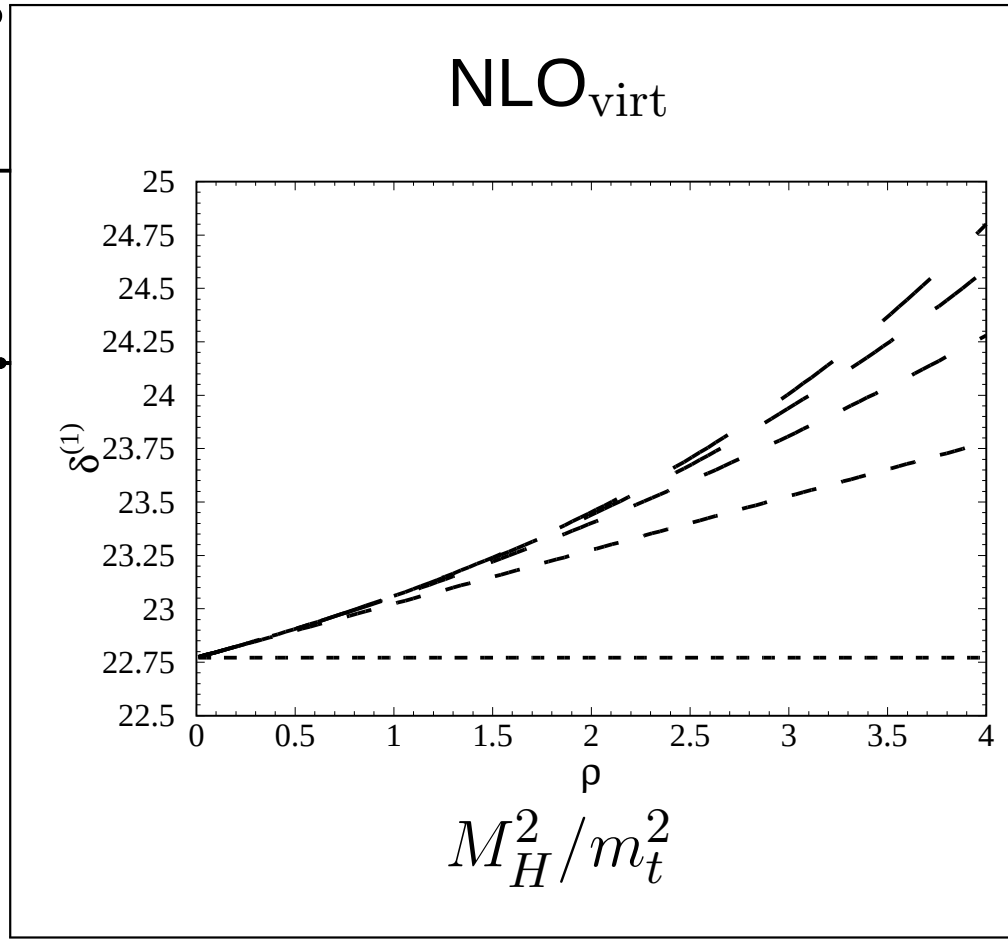


- promising:
  expansion for $m_t^2 \gg M_H^2$

$1/m_t$ corrections

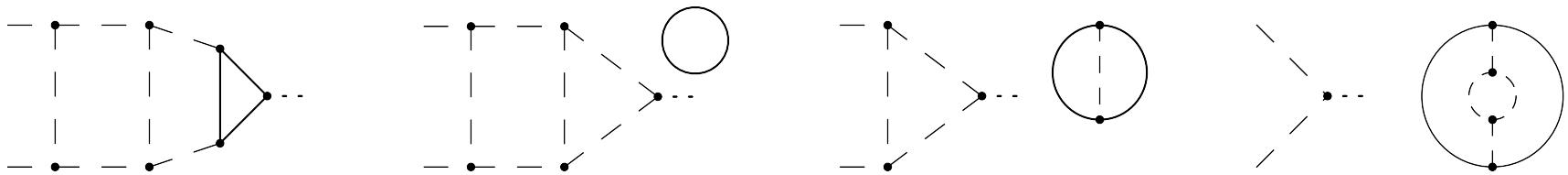
- NLO: “full” vs. “$m_t \to \infty$” $\longrightarrow$ few %
- NNLO: $1/m_t$ corrections ?
- 1. step: virtual corrections





- promising:
  expansion for $m_t^2 \gg M_H^2$

Asymptotic expansion



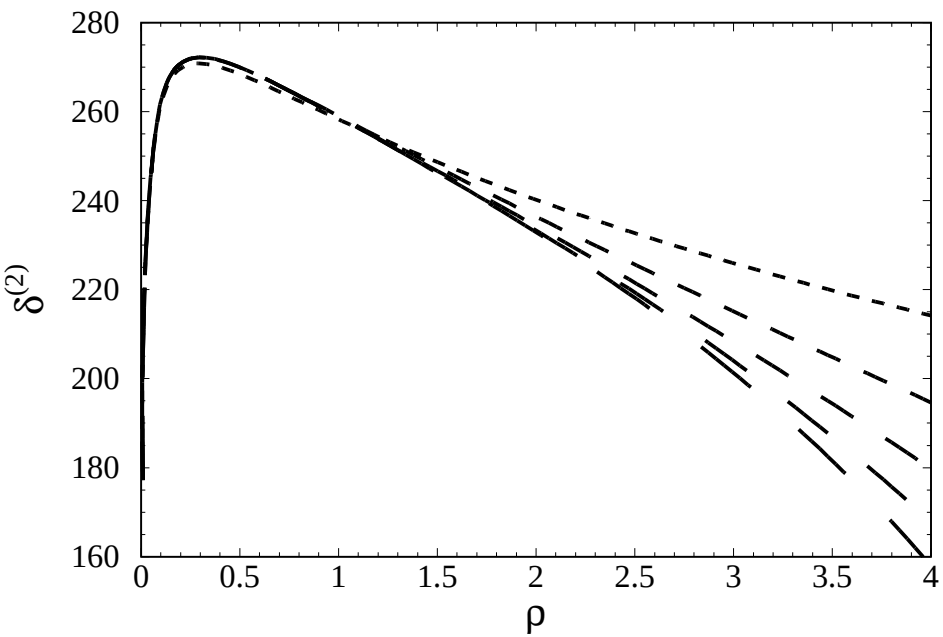
- massive 3-loop bubbles
- massless 2-loop vertices
- ≈ 660 diagrams; ≈ 1400 sub-diagrams;
- expansion up to $(M_H^2/m_t^2)^4$ [Pak,Rogal,MS'09]
- parallel calculation: [Harlander,Kemal'09]
  (expansion up to $(M_H^2/m_t^2)^2$)

Results

$$\hat{\sigma}_{ggh}^{\rm virt} = \frac{G_F\,\alpha_s^2}{288\sqrt{2}\pi}\frac{f_0(\rho,\epsilon)}{(1-\epsilon)}\,\delta(1-x)\left(1+\frac{\alpha_s}{\pi}\,\delta^{(1)}+\left(\frac{\alpha_s}{\pi}\right)^2\delta^{(2)}+\ldots\right)$$

[Pak,Rogal,MS'09]

$\delta^{(2)}$



M_H^2/m_t^2

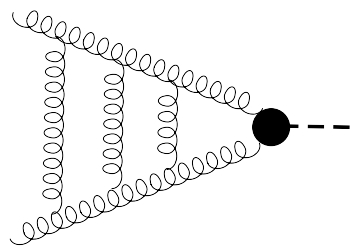
Checks:

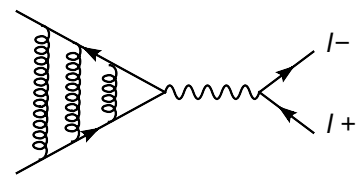
- $1/m_t^0$: agreement with [Chetyrkin,Kniehl,MS'97; Harlander'00]
- transversal structure
- general ξ up to $(M_H^2/m_t^2)^2$
- "hard part": agreement with $\Gamma(H\to gg)$ [Schreck,MS'07]
- agreement with [Harlander,Kemal'09]

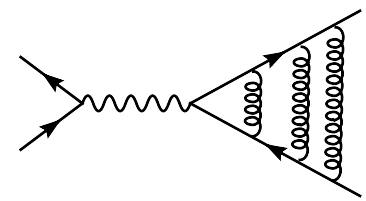
Summary

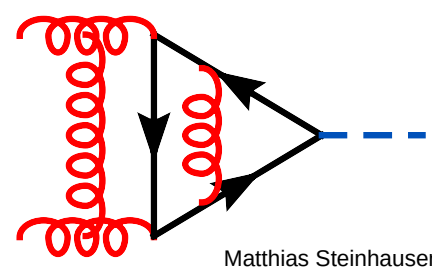
I. Massless quark and gluon form factor

- 3-loop corrections for F_q and F_g
- 3 most complicated MIs
- applications: NNNLO contribution to
  - $gg \to H$
  - DY
  - $e^+e^- \to 2$ Jets







II. $gg \to H$ @ NNLO beyond $m_t \to \infty$: virtual corrections

- virtual 3-loop corrections up to order $(M_H^2/m_t^2)^4$