Hadronic Form Factors and V_{CKM} determination

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Introduction

- quark weak current in SM:

$$j_\mu^W = \overline{U}_L V_{CKM} \gamma_\mu D_L \qquad \overline{U} = (\bar{u}, \bar{c}, \bar{t}), \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

quark e.m. current:

$$j_\mu^{em} = e_u \bar{U} \gamma_\mu U + e_d \bar{D} \gamma_\mu D$$

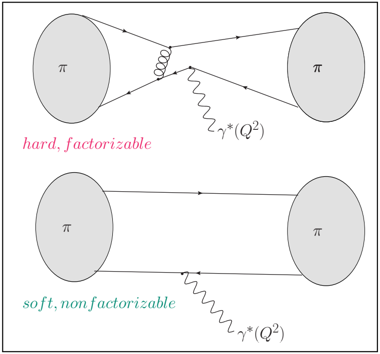
- both currents generate hadronic transitions, e.g.,

$$\boxed{\langle \pi(p) | j_\mu^W | D(p+q) \rangle = V_{cd} f^+_{D\pi}(q^2)(2p+q)_\mu + ...,}$$

$$\boxed{\langle \pi(p+q) | j_\mu^{em} | \pi(p) \rangle = F_\pi(Q^2)(2p+q)_\mu} \qquad Q^2 = -q^2$$

- hadronic form factors, weak: $f^+_{D\pi}(q^2)$, $f^+_{B\pi}(q^2)$, ...
  and e.m.: $F_\pi(Q^2)$,...

Form factors of pseudoscalar mesons



replace:

light quark	$\Rightarrow$	c, b
j_μ^{em}	$\Rightarrow$	j_μ^{W}
large Q^2	$\Rightarrow$	$q^2 \simeq 0$
		$(m_{c,b} - large)$

- $F_\pi(Q^2)$ and $f^+_{D\pi}(q^2)$, $f^+_{B\pi}(q^2)$ are similar objects from QCD point of view,
- a universal method to calculate these form factors ?

In what follows:

- New calculation of $D \to \pi$ and $D \to K$ form factors from QCD light-cone sum rules (LCSR):

  A.K.,Ch.Klein,Th.Mannel,N.Offen, 0907.2842 [hep-ph]

  - recent very accurate CLEO Collaboration data:

    $D \to \pi(K) e \nu_e$: *[arXiv:0906.2983[hep-ex]]*

    $D \to \mu\nu_\mu$ *[arXiv:0806.2112 [hep-ex]]*

  - $|V_{cd}|$ and $|V_{cs}|$ determination, decreasing the theory error
  - predicting the form factors at $q^2 > 0$ from analyticity
- Status of $B \to \pi$ form factor and $|V_{ub}|$
- Decay constants f_B and $f_{D_{(s)}}$
- LCSR for pion form factors:

  $F_\pi(Q^2)$ and $F_{\gamma\pi}(Q^2)$ confronting exp. data

  (recent BABAR measurement of $F_{\gamma\pi}(Q^2)$)

Light-cone sum rules (LCSR)

- $D \to \pi$: correlator of $j_\mu = \overline{d}\gamma_\mu c$ and $j_5 = m_c \overline{c} i\gamma_5 u$

$$\int d^4x e^{iqx}\langle\pi(p)|T\{j_\mu(x)j_5(0)\}|0\rangle = \frac{\langle\pi|j_\mu|D\rangle\langle D|j_5|0\rangle}{m_D^2-(p+q)^2} + \sum_h \frac{\langle\pi|j_W|h\rangle\langle h|j_5|0\rangle}{m_h^2-(p+q)^2}$$

$$q^2,(p+q)^2 \ll m_c^2 \Downarrow \quad x^2 \to 0$$

$$\boxed{\sum_{t=2,3,4} C_t(x^2,m_c)\langle\pi(p)|O_t(x,0)|0\rangle} \Leftarrow \text{OPE near LC}$$

- $\langle\pi(p)|O_t(x,0)|0\rangle$ - pion light-cone distribution amplitudes,

  twist /Fock-state expansion:

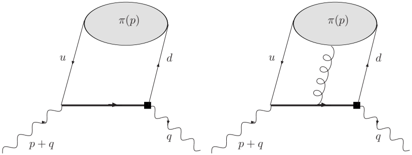
  $O_2 = \bar{u}(x)\gamma_\mu\gamma_5 d(0) \Rightarrow \varphi_\pi(u)\oplus tw4,$

  $O_3 = \bar{u}(x)i\gamma_5 d(0) \Rightarrow \varphi_{3\pi}(u)$, ...

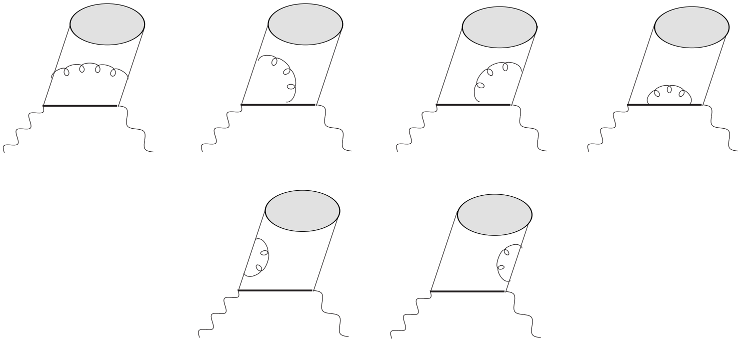
  $\widetilde{O}_3 = \bar{u}(x)G_{\mu\nu}(vx)\gamma_5 d(0) \Rightarrow \Phi_{3\pi}(\alpha_1,\alpha_2)$, $\widetilde{O}_4 = \dots$

Diagrams of the OPE

LO, including 3-particle DA's $\Rightarrow$



NLO, with collinear factorization:



virtual *b* or *c* quark with finite $\overline{MS}$ mass

Practical use of LCSR

- matching OPE and dispersion relation

  at large $|(p+q)^2| \to M^2$ and using quark-hadron duality:

$$F_{OPE}((p+q)^2, q^2) = \sum_{t=2,3,4} \int du\ C_t((p+q)^2, q^2, u) \otimes \varphi_\pi^t(u)$$

$$= \frac{m_D^2 f_D f_{D\pi}^+(q^2)}{m_D^2 - (p+q)^2} + \int\limits_{s_0^D}^{\infty} ds \frac{\text{Im} F_{OPE}(s, q^2)}{s - (p+q)^2}$$

- calculation at finite m_c, $\quad M^2 \sim m_c\tau$, $\tau \gg \Lambda_{QCD}$

  (Λ_{QCD}/m_b) and/or (Λ_{QCD}/τ) suppression of higher twists

Input

- c-quark mass:

  $\bar{m}_c(\bar{m}_c) = (1.29 \pm 0.03)\,\mathrm{GeV}$,

  [J. H. Kühn, M. Steinhauser and C. Sturm,(2007)];

  [R. Boughezal, M. Czakon and T. Schutzmeier (2006)]

  (the talk by Kühn)

- light quark masses:

  $m_s(\mu = 2\,\mathrm{GeV}) = (98 \pm 16)\,\mathrm{MeV}$, $m_{u,d} \sim 0$

  (QCD SR in $O(\alpha_s^4) \oplus$ ChPT, Leutwyler relations)

- pion and kaon DA's: $\varphi^{(t)}_{\pi(K)}(u)$

  $\varphi^{(2)}_{\pi}(u)$, LCSR for $B \to \pi$, fitting the shape to exp.

  [G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen (2007)

  $\varphi^{(2)}_{K}(u)$,asymmetry- *[K.Chetyrkin, A.K., A.Pivovarov (2007)]*

  t=3,4 DA's: *[P.Ball, V.Braun, A.Lenz (2006)]*

- f_D- measured by CLEO (assuming $|V_{cd}| = |V_{us}|$)
  and/or determined from QCD (SVZ) sum rule

Universality of the method

- LCSR reproduces *both* "soft" and "hard" parts of a form factor: the soft contribution dominates (no α_s!)
- $c \to b$ in the correlator
  $\Rightarrow$ LCSR for $B \to \pi, K$ form factors
- $c \to u, d$, large Q^2
  $\Rightarrow$ LCSR for the pion form factor $F_\pi(Q^2)$

  reproduces the QCD asymptotics $\sim \alpha_s/Q^2$ at $Q^2 \to \infty$

  [V. Braun, I. Halperin(1997)]; [V. Braun, A.K., M.Maul (2000)],

  [J.Bijnens, A.K. (2002)]

Limitations of the method

- only the correlator is calculable,
  no "direct access" to the hadronic matrix element
  (the same in lattice QCD)
- accuracy (estimated at $\sim \pm 15\%$ level) limited by:
  - truncated twist expansion
  - variation of input parameters, scales
  - uncertainty of $\varphi_\pi(u)$ (Gegenbauer moments)
  - quark-hadron duality approximaiton $\rightarrow s_0^D$

  (controlled by the m_D calculation)
- the region of accessible q^2 is restricted:

  $f^+_{D\pi}(q^2)$ at $q^2 \leq 0$, $f^+_{B\pi}(q^2)$ at $q^2 \ll (m_B - m_\pi)^2$,

  $F_\pi(Q^2)$ at $Q^2 = -q^2 \geq 1$ GeV^2

$D \to \pi, K$ form factors at $q^2 = 0$

Method	[Ref.]	$f^+_{D\pi}(0)$	$f^+_{DK}(0)$
Lattice QCD	[APE(2001)]	$0.57 \pm 0.06 \pm 0.02$	$0.66 \pm 0.04 \pm 0.01$
	[Aubin et al (2005)]	$0.64 \pm 0.03 \pm 0.06$	$0.73 \pm 0.03 \pm 0.07$
	[QCDSF(2009)]	$0.74 \pm 0.06 \pm 0.04$	$0.78 \pm 0.05 \pm 0.04$
LCSR	[A.K. et al. (2000)]	0.65 ± 0.11	$0.78^{+0.2}_{-0.15}$
	[P. Ball (2006)]	0.63 ± 0.11	0.75 ± 0.12
	this work	$0.67^{+0.10}_{-0.07}$	$0.75^{+0.11}_{-0.08}$

$$\frac{f^+_{D\pi}(0)}{f^+_{DK}(0)} = 0.88 \pm 0.05$$

some uncertainties cancel

Determination of $|V_{cd}|$

- LCSR predicts the product, $[f_D f_{D\pi}(0)]_{LCSR} = 137^{+19}_{-14}$ MeV
- CLEO data :

$$\frac{dBR}{dq^2}(D \to \pi l \nu_l) \oplus \{\text{fit of the } q^2\text{-bins}\}$$

$$\Rightarrow f_{D\pi}(0)|V_{cd}| = 0.150 \pm 0.004 \pm 0.001\,,$$

$$BR(D \to l\nu_l) \Rightarrow f_D|V_{cd}| = 46.5 \pm 2.0 \text{ MeV}$$

(CLEO quotes f_D assuming $|V_{cd}| = |V_{us}|$)

- multiply two exp. numbers and divide by LCSR prediction:

$$|V_{cd}| = 0.225 \pm [0.005]_{exp1} \pm [0.003]_{exp2}{}^{+0.016}_{-0.012}\,,$$

theory (LCSR) error is effectively halved !

- comparison: CLEO $\oplus$ lattice:
  $|V_{cd}| = 0.234 \pm 0.007 \pm 0.002 \pm 0.025$

Determination of $|V_{cs}|$

- CLEO data:

$$\frac{f_{D\pi}(0)|V_{cd}| = 0.150 \pm 0.004 \pm 0.001}{f_{DK}(0)|V_{cs}| = 0.719 \pm 0.006 \pm 0.005},$$

- from the predicted ratio of $D \to \pi$ and $D \to K$ form factors

$$\frac{|V_{cd}|}{|V_{cs}|} = 0.236 \pm [0.006]_{exp} \pm [0.003]_{exp} \pm 0.013\,,$$

- comparison, CLEO $\oplus$ lattice:

$$\frac{|V_{cd}|}{|V_{cs}|} = \frac{0.234 \pm 0.007 \pm 0.002 \pm 0.025}{0.985 \pm 0.009 \pm 0.006 \pm 0.103}$$

Accessing the form factors in the semileptonic region

- Can we obtain the $D \to \pi(K)$ form factors at $0 < q^2 < (m_D - m_{\pi(K)})^2$?
  where LCSR is not applicable !
- $f^+_{D\pi}(q^2)$ is analytic in q^2 , dispersion relation:

$$f^+_{D\pi}(q^2) = \frac{f_{D^*} g_{D^* D\pi}}{m^2_{D^*} - q^2} + \frac{1}{\pi} \int\limits_{(m_D+m_\pi)^2}^{\infty} ds \frac{\mathrm{Im} f^+_{D\pi}(s)}{s - q^2}$$

(no subtractions due to the QCD asymptotics)

- $f^+_{D\pi}(q^2)$ real at $q^2 < m^2_{D^*}$, pole at $q^2 = m^2_{D^*}$,
  branch points (and poles) at $q^2 > (m_D + m_\pi)^2$
- first possibility:
  fit the LCSR form factor at $q^2 \leq 0$ to disp. relation
  and access $q^2 \leq 0$ via analytic continuation
  (need model-dependent parameterization of the integral)

Conformal mapping

- map the complex q^2-plane onto $|z| < 1$ in the z-plane:

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \quad t_+ = (m_D + m_\pi)^2, t_0 < t_+$$

[N. Meiman (1963)]; [B.Ioffe, B.Geshkenbein (1963)], [S.Okubo (1971)]; [C.Bourrely, B.Machet, E. de Rafael (1981)]

- many applications to $B \to \pi$ and other form factors:
  [C.G.Boyd, B.Grinstein, R.Lebed (1995)],
  [L.Lellouch (1996)],..., [T.Becher, R.Hill (2006)],..
- combined with perturbative QCD bounds
  (from the unitarity for the 2-point correlation function)
- bounds not very restrictive, $|z^{D\pi}| \leq 0.22$, $|z^{DK}| \leq 0.09$,
  $\Rightarrow$ a simple Taylor expansion near $z = 0$ is sufficient

Series paramaterizations

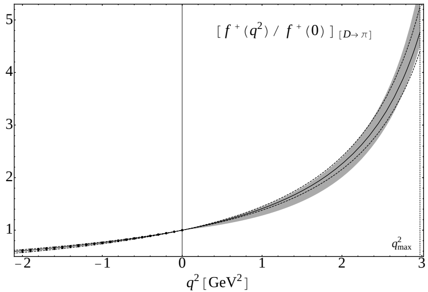
- The last (and simplest) version
  [C. Bourrely, I. Caprini, L. Lellouch (2008)]
  a power expansion with $\sim 3, 4$ parameters

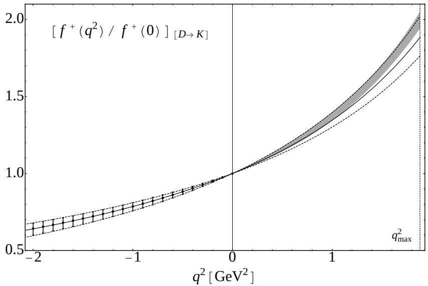
$$f^{+}_{D\pi}(q^2) = \frac{1}{1 - q^2/m^2_{D^*}} \sum_{k=0}^{k_{max}} b_k \Big(z(q^2, t_0) \Big)^k$$

  D^* the vector-meson pole near threshold

- we calculate $f^{+}_{D\pi(K)}(q^2)$ at $q^2 \leq 0$ from LCSR,
  then fit to z-expansion and continue over z
  $\Rightarrow$ $f^{+}_{D\pi(K)}(q^2)$ at $0 \leq q^2 \leq (m_D - m_{\pi(K)})^2$

$D \to \pi, K$ form factor shapes





$f^+_{D\pi}(q^2)/f^+_{D\pi}(0)$ $\qquad$ $f^+_{DK}(q^2)/f^+_{DK}(0)$

- LCSR at $q^2 \leq 0$ (points) $\oplus$ fit to series (3 param.)
  $\Rightarrow$ form factors at $q^2 \geq 0$ (solid), with uncertainties (dashed).
- CLEO fit to the shape (shaded)

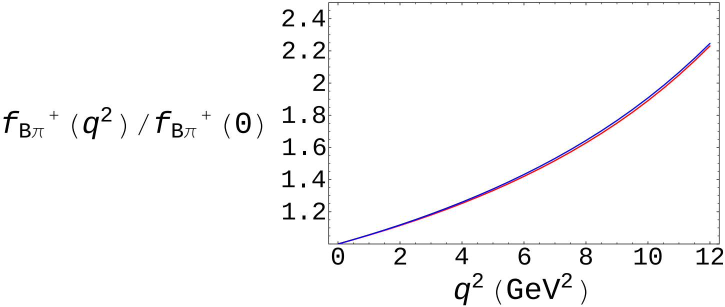
$B \to \pi$ form factor from LCSR

- last update, $f^+_{B\pi}(q^2)$ at $0 < q^2 \leq 10$ GeV^2 ,

  [G.Duplancic, A.K., B.Melic, Th.Mannel, N.Offen (2007)

$$f^+_{B\pi}(0) = 0.26^{+0.04}_{-0.03}$$

- with f_B from two-point QCD sum rules (exp. f_B still with a larger error)
- fitting the shape to the *BABAR* data to fix φ_π (Gegenbauer moments)



$$\varphi_\pi(u, 1\text{GeV}) = 6u(1-u)\Big(1 + a_2 C_2^{3/2}(2u-1) + a_4 C_4^{3/2}(2u-1)\Big)$$

$a_2 = 0.16 \pm 0.01$ $a_4 = 0.04 \pm 0.01$

Recent $|V_{ub}|$ determinations from $B \to \pi l \nu_l$

[ref.]	$f^+_{B\pi}(q^2)$ calculation	$f^+_{B\pi}(q^2)$ input	$\lvert V_{ub}\rvert \times 10^3$
Okamoto et al. '05	lattice ($n_f = 3$)	-	3.78±0.25±0.52
HPQCD '06	lattice ($n_f = 3$)	-	3.55±0.25±0.50
Flynn et al '07	-	lattice $\oplus$ LCSR	$3.47 \pm 0.29 \pm 0.03$
Ball, Zwicky '04	LCSR	-	$3.5 \pm 0.4 \pm 0.1$
DKMMO '07	LCSR	-	$3.5 \pm 0.4 \pm 0.2 \pm 0.1$
Bourrely, Caprini, Lellouch '08	-	lattice$\oplus$ LCSR	3.54 ± 0.24

- Unitarity triangle fit: $|V_{ub}| = (3.5^{+0.15}_{-0.14}) \times 10^{-3}$

[CKM Fitter , Moriond (2009)]

B and $D_{(s)}$ decay constants [in MeV]

a sample of recent results:

method	f_B	f_D	f_{D_s}
exp. $\oplus$ CKM	$(242 \pm 28)\frac{3.99\times 10^{-3}}{V_{ub}}$ *[Belle '08]*	$205.8 \pm 8.5 \pm 2.5$ *[CLEO'08], $V_{cd} = V_{us}$*	$259.5 \pm 6.6 \pm 3.1$ *[CLEO'09], $V_{cs} = V_{ud}$*
lattice	190 ± 13 *[HPQCD,'09]*	207 ± 4 *[HPQCD,UKQCD '08]*	241 ± 3 *[HPQCD,UKQCD '08]*
QCD SR	210 ± 19 *[Jamin-Lange '01]*	-	
	206 ± 20 *[Penin-Steinhauser'01]*	195 ± 20 *[Penin-Steinhauser'01]*	
		203 ± 20 *[Narison '02]*	235 ± 24 *[Narison '02]*
OPE bound	-	<230	<270

- f_{B_s}/f_B, SR in agreement with lattice QCD
- still some tension of exp. vs lattice f_{D_s} (and vs the bound)
- already some tension in f_B ?

OPE bounds for f_{D_s} and f_D

[A.K., hep/ph-0812.3747]

- Correlator of two $j_5(x) = (m_c + m_s)\bar{s}(x) i\gamma_5 c(x)$,

$$\Pi(q^2) = \int d^4x\, e^{iqx} \langle 0 | T\{ j_5(x) j_5^\dagger(0)\} | 0 \rangle = \frac{f_{D_s}^2 m_{D_s}^4}{m_{D_s}^2 - q^2} + \sum_h \frac{\langle 0 | j_5 | h \rangle \langle h | j_5^\dagger | 0 \rangle}{m_h^2 - q^2}$$

$q^2 \ll m_c^2 \Downarrow \quad x \to 0 \qquad\qquad \langle 0 | j_5 | D_s \rangle = m_{D_s}^2 f_{D_s}$

$$\boxed{\sum_{d=0,3,4,..} C_d(q^2, m_c, \alpha_s) \langle 0 | O_d(0) | 0 \rangle} \quad \Leftarrow \text{OPE}$$

$d \neq 0$, $\langle 0 | O_d | 0 \rangle$- vacuum condensates,

$O_3 = \bar{q}q$, $O_4 = G_{\mu\nu} G^{\mu\nu}$, ...

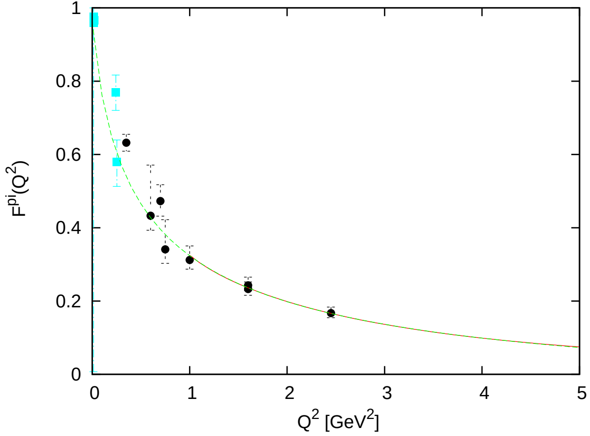
- ▶ calculate the (Borelized) correlator
- ▶ input: quark masses, α_s and condensates
- ▶ use the positivity of the hadronic sum

  no quark-hadron duality involved

- bound for $f_{B_{(s)}}$ is not constraining

The pion e.m. form factor [preliminary]

- LCSR calculated at $Q^2 = 1.0 - 5.0$ GeV^2 (solid), fit to series (z)-param. (dashed) $\to Q^2 < 1.0$ GeV^2
  input: pion DA's used for $B \to \pi$, $D \to \pi$



data: Jlab [2008] (points), FNAL (points) [Amendolia et al (1986)]

$\gamma \to \pi$ form factor

- The $\gamma^*(q)\gamma^*(k) \to \pi^0$ amplitude from light-cone OPE , both $Q^2 = -q^2$ and $|k^2|$ large:

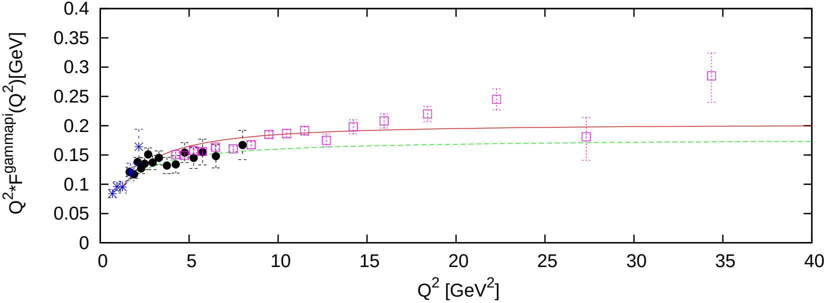
$$F^{\gamma^*\pi}(Q^2, k^2) = \int\limits_0^1 du \frac{\varphi_\pi(u)}{uQ^2 + (1-u)|k^2|} + O(\alpha_s) + O\left(\frac{1}{Q^{4,6,.}}\right)$$

- exp. data on two-photon processes available only at $k^2 = 0$ $F^{\gamma^*\pi}(Q^2, 0) \equiv F^{\gamma\pi}(Q^2)$, the form factor
- at $k^2 = 0$ the above formula is not directly applicable the real photon is a hadronlike object
- $F^{\gamma\pi}(Q^2)$ obtained from matching with the dispersion relation in k^2

$$F^{\gamma^*\pi}(Q^2, k^2) = \frac{F_{\rho\pi}(Q^2)}{m_\rho^2 - k^2} + \frac{1}{\pi}\int_{s_0}^{\infty} \frac{\text{Im} F^{\gamma^*\gamma^*\pi}(Q^2, s)}{s - k^2} \qquad (1)$$

[A.K. (1999)], ..., [Mikhailov,Stefanis, hep-ph 0905.404]

$Q^2 F^{\gamma\pi}(Q^2)$ [preliminary]



"default" $\varphi_\pi(u)$ (solid), asymptotic (dashed)

data: CELLO (crosses), CLEO (points), BABAR '09 (squares)

- at $Q^2 < 10$ GeV^2 good agreement
- BABAR data indicating a growth in Q^2 ?
- $O(\alpha_s[logQ^2]^2)$ to be resummed
- $\varphi_\pi(u) = const$ "overshoots" the data and destroys agreement with $F_\pi(Q^2)$

Summary

- new $|V_{cd}|$ and $|V_{cs}|$ determination from QCD light-cone sum rules
- agreement with lattice QCD, uncertainties comparable
  - lattice: future goal of $\sim 1 \div 2\%$ accuracy
  - LCSR, QCD SR accuracy will remain at most 10%
- future aim: input and duality uncertainties
- analyticity and matching of dispersion relation and/or series parameterization provides a useful tool to go beyond the accessible regions of q^2
- the method (with universal input) works for various form factors: $B \to \pi$, $D \to \pi, K$, F_π^{em} , $F_{\gamma\pi}$,
  deviation from BABAR data at large Q^2 deserves a careful study