On discrete Minimal Flavour Violation *

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Low energy constraints

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Note added:

In the (final) paper **0908.4182[hep-ph]** the physics (as well as some of the group theory) is much more elaborated and the conclusions are rather more optimistic.

Minimal Flavour Violation

- **Yukawa** → 0 then largest global symmetry <u>commuting</u> SM gauge groups:
  $G_F = G(\text{quark})_F \otimes G(\text{lepton})_F$ --- $G(\text{quark})_F = SU(3)_Q \times SU(3)_{UR} \times SU(3)_{DR} \times U(1)_{UR} \times U(1)_{DR}$ ..... *

  Chivukula & Georgi'87

- <u>Let</u> **Yukawa** <u>formally</u> transform $Y_D \sim (3^*,1,3)_{0,-1}$ .... G_F symmetry restored

> **MFV** effective theory (of higher dimensional operators)
> invariant under global G_F and CP with O(1) coefficients.

D'Ambrosio, Giudice, Isidori & Strumia '02

- Symmetry allows relax CP option (MSSM generically has CP phases e.g. Isidori, Mescia Smith, Trine'06)
- G_F-symmetry broken by Yukawa --- how is the symmetry broken ?
- Spontaneously? ⇒ Yukawa spurions VEV $\langle Y_U \rangle \neq 0$
  ⇒ 3x8+2x1 = 26 (massless) Goldstone bosons .. *where are they, some high scale?*

*two U(1) factors further clarified Feldmann, Mannel, Jung '09

Discrete Minimal Flavour Violation

- SSB of discrete symmetries does not lead to Goldstone bosons ⇒ discrete flavour symmetry? *

Does discrete symmetry provide sufficient protection for flavour?

Formulation:

1. $G(\text{quark})_F \rightarrow D(\text{quark})_F = \mathbf{D3_Q \times D3_{UR} \times D3_{DR} \times D1_{UR} \times D1_{DR}}$ ($D3 \subset SU(3)$, $D1 \subset U(1)$)
2. Specify the three dimensional **representation** of D3...
3. (Possibly) **embedding** of D3 into SU(3)

- How does such an effective theory look like ('model independent' approach)

⇒ study of **invariants** (~ effective operators)

* alternatively: Higgs mechanism, Axion like mechanism Albrecht et al to appear

...a few clarifications before we start

- Families into triplets U = (u,c,t) of discrete groups D_U

- In the model independent approach that we follow here the effective Lagrangian

$$L_{eff} = c_1 \boldsymbol{I_1} (U,....) + c_2 \boldsymbol{I_2} (U,....) + h.c.$$

$\boldsymbol{I}$ = Invariant under D_U , $|c_1| \sim |c_2|$ coefficients of same order

- No attempt to explain the CKM & mass hierarchies with discrete symmetry
(Many papers on the arxiv attempting to do that via scalar sector extensions
Froggat-Nielsen type mechanism popular at the end '70 and revived with tri-bi maximal mixing) *

not a theory of flavour

- Moreover no obvious relation to CMFV by Buras et al '01?

*within MFV scenario outlined Feldmann, Mannel, Jung '09

discrete (finite)groups ... generalities & examples

- Example: *three permutations* $\mathbf{S_3}$: Order $|S_3|=3!=6$ ( ),(12),(23),(13),(123),(132)
- Irreducible representations (IRREPS): |D| = ∑ |IRREPS(D)|^2 ⇒ finite many of them
- $\mathbf{A_4} = \Delta(12)$ (*even four permutation*)

popular tri-bi maximal mixing

$$|A_4| = 4!/2 = 12 = \mathbf{1}^2 + \mathbf{1'}^2 + \mathbf{1''}^2 + \mathbf{3}^2$$

3d IRREPS generated by

$$a = \begin{matrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{matrix} \qquad d = \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

Ex. Kronecker product: $\mathbf{3} \times \mathbf{3} = \mathbf{1} + \mathbf{1'} + \mathbf{1''} + \mathbf{3}_s + \mathbf{3}_a$

$\mathbf{1'} \sim (\omega^2 x_1 y_1, \omega x_2 y_2, x_3 y_3)$; $\omega = \exp(2\pi i/3)$

a: $\mathbf{1'} = \omega\ \mathbf{1'}$

- Many others than permutation groups -- any finite group can be embedded in a permutation group

Finite subgroups of SU(3)

- Classified in a classic book — Miller, Dickson, Blichfeld '1916
  Analyzed further eightfold way — Fairbairn, Fulton, Klink '64
  Further analyzed (lattice ...) — Bovier, Luling, Wyler '81
  Rescrutinized tri-bi-hype — Luhn, Nasri Ramand., '06-08

Dihedral like: $\Delta(3n^2)$ $\{\Delta(6n^2)\}$

- $Z_n \times Z_n \rtimes Z_3$ $\{S_3\}$

- largest irrep 3{6}d
  labelled (k,l){l} mod n (equivalences..)

$$d = \begin{pmatrix} \eta^{-k-l} & 0 & 0 \\ 0 & \eta^{l} & 0 \\ 0 & 0 & \eta^{k} \end{pmatrix}$$

c = similar to d

a = permutator

$(\eta \equiv \exp(2\pi i/n))$

generators .. exhibit sthg like a flavour charge

Crystallographic groups , Σ

- finite many of them
  $\Sigma(60) \sim A_5$
  $\Sigma(168) \sim PSL(2,7)$
  $\Sigma(216\varphi)$ hessian group
  $\Sigma(360\varphi)$
  φ=1,3 related center SU(3)

- used in lattice $SU(3)_{color}$ discretizations '80

Q1: Are there new invariants as compared SU(3) ?

- look d_L-sector ... (omit L,γ_μ etc, denote: $\lambda \equiv \lambda_{FC} = Y_U Y_U^\dagger$ & $\bar{d} = D$)

	MFV	ΔF	**DMFV**	ΔF
$O(\lambda^0)$	DdDd	0	$t^{ab}{}_{cd}\, d_a d_b D^c D^d$	?
$O(\lambda^1)$	DλdDd	1	t(d,d,D,D,λ)	?
$O(\lambda^2)$	DλdDλd	2	t(d,d,D,D,λ, λ)	? *

Invariants denoted ***I*** later on

*assuming D_{UR} indices can be contracted

Something useful about invariants ....

- Let **A**, **B**,**C**, ... be irreps of some group then **A** x **B** x **C** x ... = n**1** + ...
  ⇒ n ≡ number of times **1** appears = number of invariants

  (follows from **V** x **V*** = 1**1** + ... ⇔ **V** irrep)

- Example: SU(3) (**3** x **3***) x (**3** x **3***) = (**1**+ **8**) x (**1**+ **8**) = 2 **1** + 2 **8**
  Invariants: $(t1)^{ab}{}_{cd} = \delta^a{}_c\, \delta^b{}_d$ $(t2)^{ab}{}_{cd} = \delta^a{}_d\, \delta^b{}_c$ normal & Fierz

- crucial example: SU(3): $(\mathbf{3} \times \mathbf{3}^*)^4 = (\mathbf{8} \times \mathbf{8})^2 + ... = ((\mathbf{1} + \mathbf{8} + \mathbf{27})_S + (\mathbf{10} + \mathbf{10}^* + \mathbf{8})_A)^2 + ...$

  Back to D_Q : $O(\lambda^2)$ $t(d,d,D,D,\lambda,\lambda) = ((DT^a d) \times (DT^b d)) \times (tr[T^c\lambda] \times tr[T^d\lambda]) + ...$
  need symmetric part ... need **27** D_Q irrep in order not to generate a new invariant!

- 1. **Dihedral groups** $\Delta(3n^2)$, $\Delta(6n^2)$ max 3,6d irrep ⇒ out

  2. **Crystallographic groups** .. look at charactertables reveals there is none
  (N.B. $27^2 = 729$ almost saturates the largest group (3x360=1080) ...)

⇒ *Answer: yes there are new invariants!*

Q2: Relation between ΔF and O(λ) and types (in flavour basis)

DMFV	ΔF
$_d d_a d_b D^c D^d$?	?
(d,d,D,D,λ)?	?
t(d,d,D,D,λ, λ)	?

- ΔF =2 $O(\lambda^0)$: Σ(60) ~ A_5 , $I_0 = D_3 d_2 \, D_3 d_2$ + permutations ('too wild')

- ΔF = a requires $O(\lambda^a)$: Σ(168) ~ PSL(2,7) **3** x **3*** = **1**+ **8** like SU(3) but **6**=**6*** unlike SU(3)

  $I_2 = D_1 D_1 \, \lambda_{21} \, \lambda_{33} \, d_2 \, d_2$ + permutations complete anarchy in transitions

⇒ *seems crystallographic groups violate flavour at some point ...*

- $\Delta(3n^2)$, $\Delta(6n^2)$: If choose irrep s.t. no real 3d irrep are generated then ΔF = a requires $O(\lambda^a)$:

  Examples: $O(\lambda^0)$ $I = D_1 d_1 \, D_2 d_2$ + permutations
  $O(\lambda^2)$ $I = (D_1 \lambda_{12} d_2)^2$ + permutations

⇒ *dihedral groups respect flavour*

flavour charge is active (in flavour basis ....)

Q3: Going to the mass basis

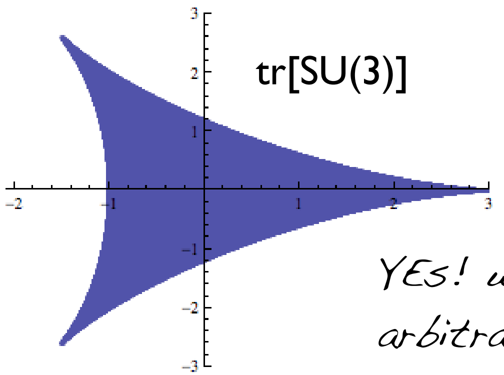
- In going to the mass basis we use $U(3)_{UL} \times U(3)_{DL} \times U(3)_{UR} \times U(3)_{UR}$
- The new invariants are constant under $D3_Q \times D3_{UR} \times D3_{DR}$ not
  $\Rightarrow$ diagonaliszation matrices $\mathbf{L_U, (L_D)\ R_U, R_D,}$ become <u>obersvable</u>

Only effect on physics in SM: $V_{CKM} = (L_U)^\dagger L_D$

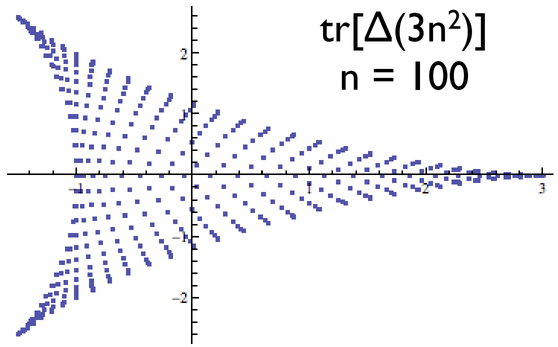
- A preferred basis has been chosen -- by whom? By us? What is that ?

A scandal!

- Can we find an embedding of say $D_{UR} \subset SU(3)$ s.t. $\mathbf{R_U} \in \mathbf{D_{UR}}$?
  Embedding ($A \in SU(3)$) $D_{UR}: \rightarrow \mathbf{A D_{UR} A^{-1}} \approx \mathbf{R_U}$, necessary condition $tr[D_{UR}] \approx tr[R_U]$



YEs! we can come arbitrarily close



If ....

- Choose embedding (try to enforce MFV ....) $L_U \approx I$, $L_D = V_{CKM}$ (or vice versa) $R_U \approx I$, $R_D \approx I$

- Recall: $O(\lambda^0)$ $\boldsymbol{I_0}$ $(\Delta(3n^2),\{\Delta(6n^2)\}) = D_1 d_1\ D_2 d_2$ + permutations
  $O(\lambda^2)$ $\boldsymbol{I_2}$ $(\Delta(3n^2),\{\Delta(6n^2)\}) = (D_1 \lambda_{12} d_2)^2$ + permutations

- $\boldsymbol{I_2}$ ok, but $\boldsymbol{I_0} = D\,(V^*_{11}\,V_{12})\,s\,D\,(V^*_{12}\,V_{22})\,s$

  $\Rightarrow \Delta S = 2$ at second order in Wolfenstein parameter !!

$\Rightarrow$ *leaves $\Sigma(168) \sim PSL(2,7)$ as the better candidate .....*

Epilogue

- Discrete MFV appears difficult in the model independent approach

- The problems raised here might serve as a guidance for a concrete model

- How would this look like in DMFV-MSSM? $(m_{Q(soft)}{}^2)^b{}_c = a_1\, \delta^b{}_c + t_1\, t^b{}_c + \ldots$

'discrete tensor'

- Also possible to work with more complicated embeddings of direct products..... for the future

Thanks for you attention!

& Christoph Luhn, Gino Isidori, Christopher Smith, Thorsten Feldmann, ... for discussion