Flavour symmetries and SUSY soft-breaking at LHC

Oscar Vives

with : L. Calibbi, J. Jones-Perez, A. Masiero, J. Park, W. Porod

Kazimierz, 25 July 2009

TIMESONLINE

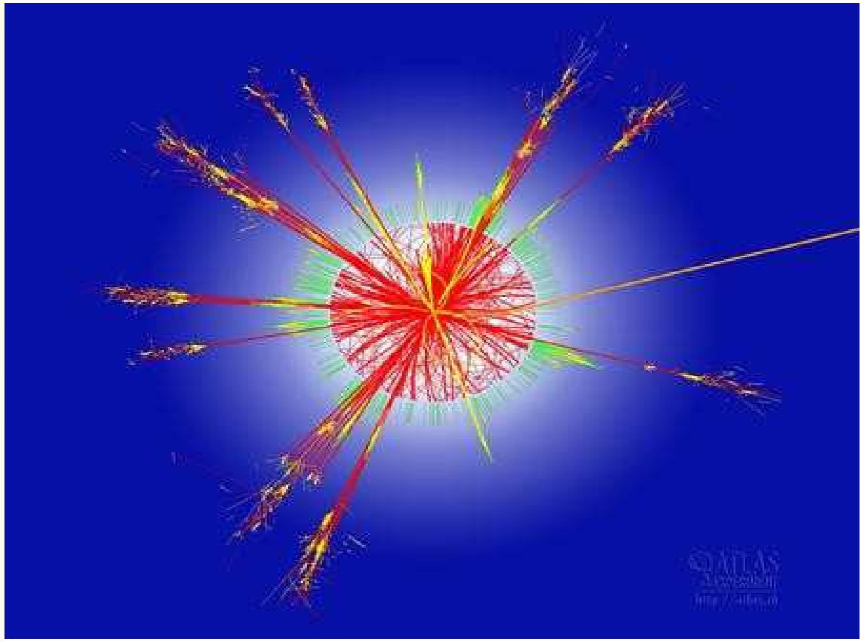
News Comment Business Money Sport Life & Style Travel Driving Arts&Ents Archive Our Papers

UK News World News Politics Environement Wheather Tech & Web Video Photo Galleries Topics Mobile

SEARCH Times Online

UK Edition

LATEST NEWS



EXCLUSIVE :

LHC finds ...

Maybe you are thinking of . . . SUSY, but, so far, most of the analysis have been made in the CMSSM : complete universality, 5 parameters.

Do you **really** believe Nature has chosen CMSSM??

Regarding flavour, it is natural to expect non-trivial flavour structures in soft-breaking sector, in analogy to Yukawas

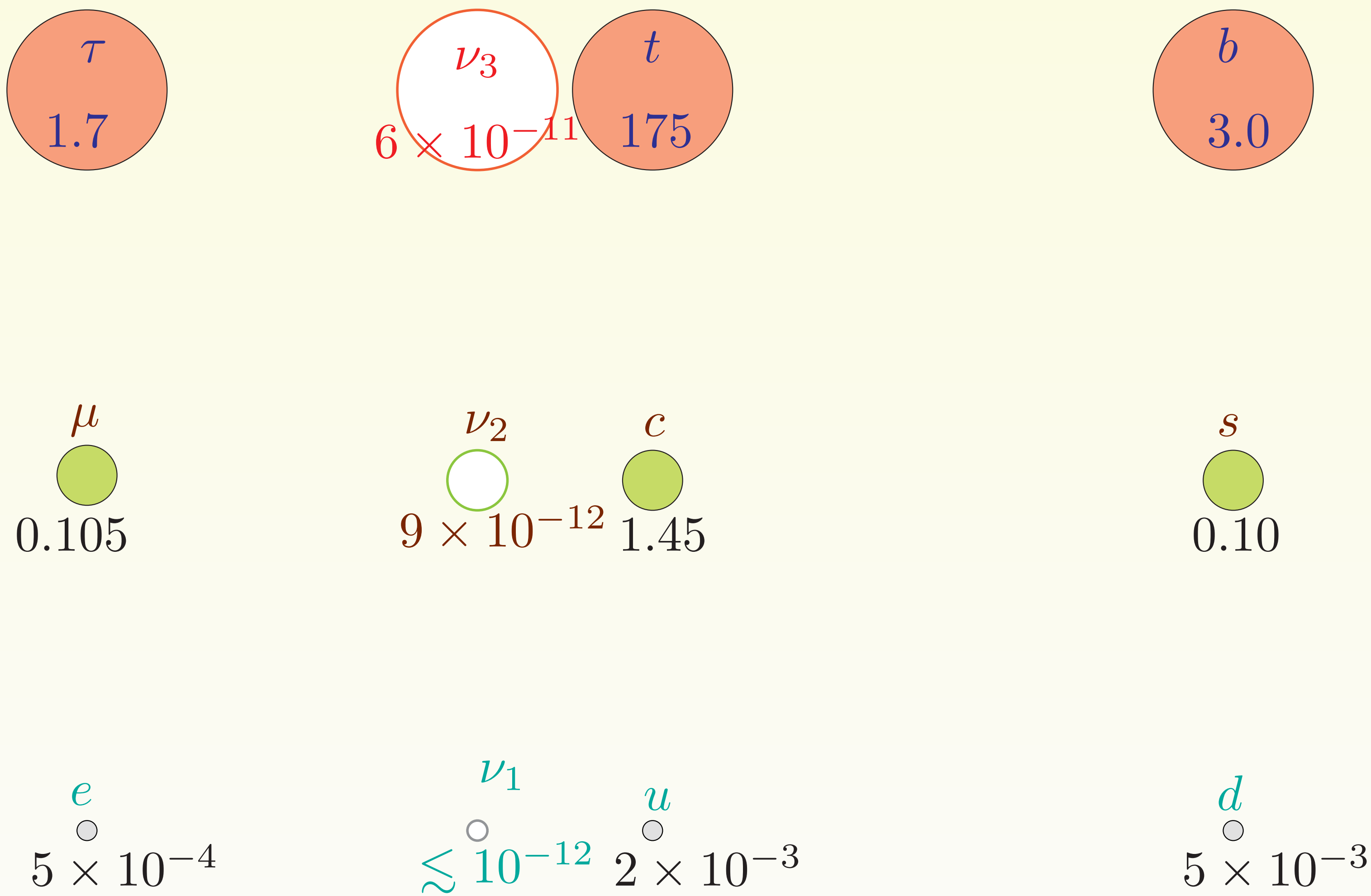
$\Longrightarrow$ It is time we consider **flavoured MSSM**!!!

Flavour physics: who ordered that??

- 3 families with identical gauge quantum numbers.
- Strong hierarchy between generations.
- Small quark, large lepton mixing angles.

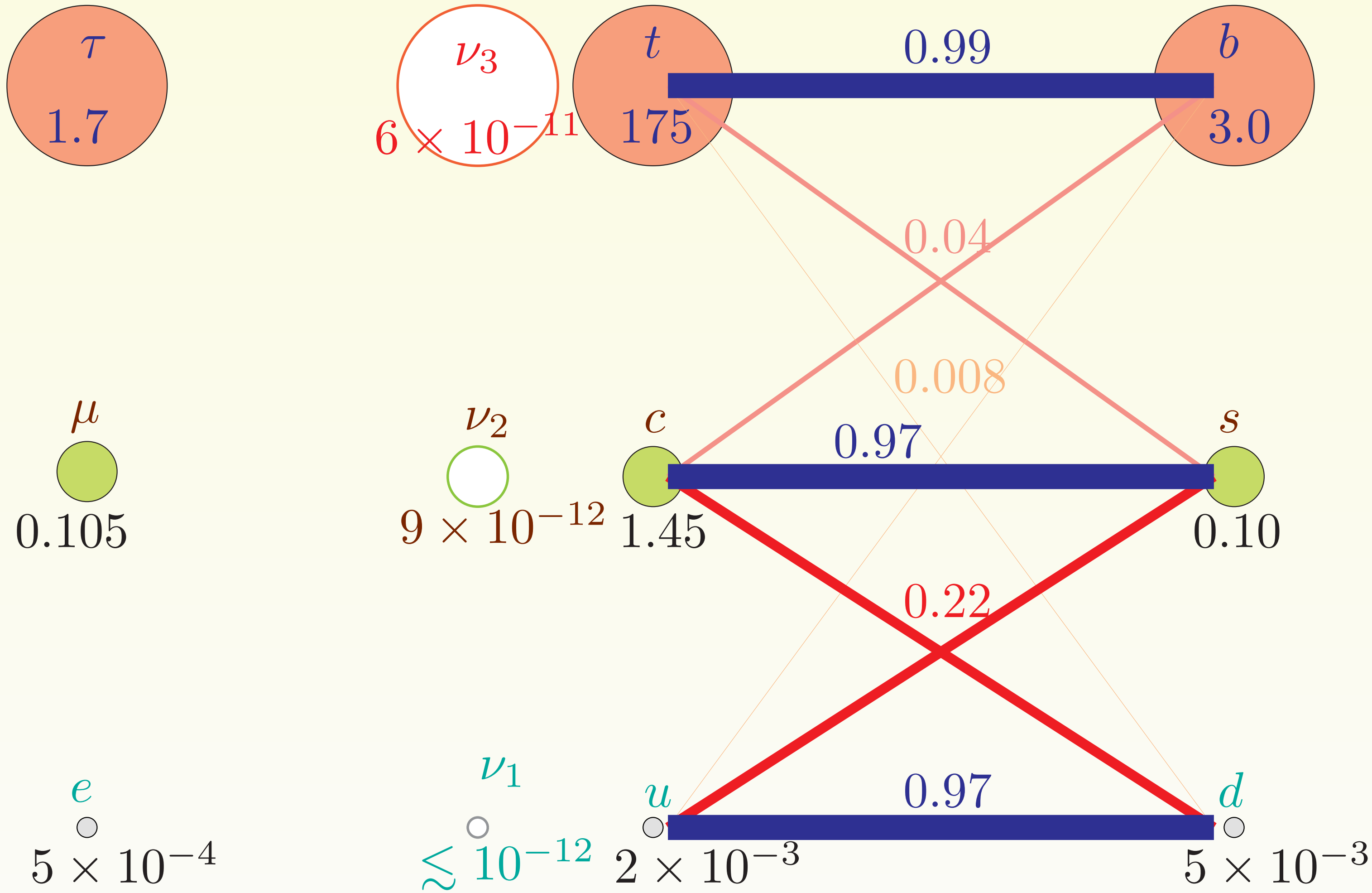
Flavour physics: who ordered that??

- 3 families with identical gauge quantum numbers.
- Strong hierarchy between generations.
- Small quark, large lepton mixing angles.



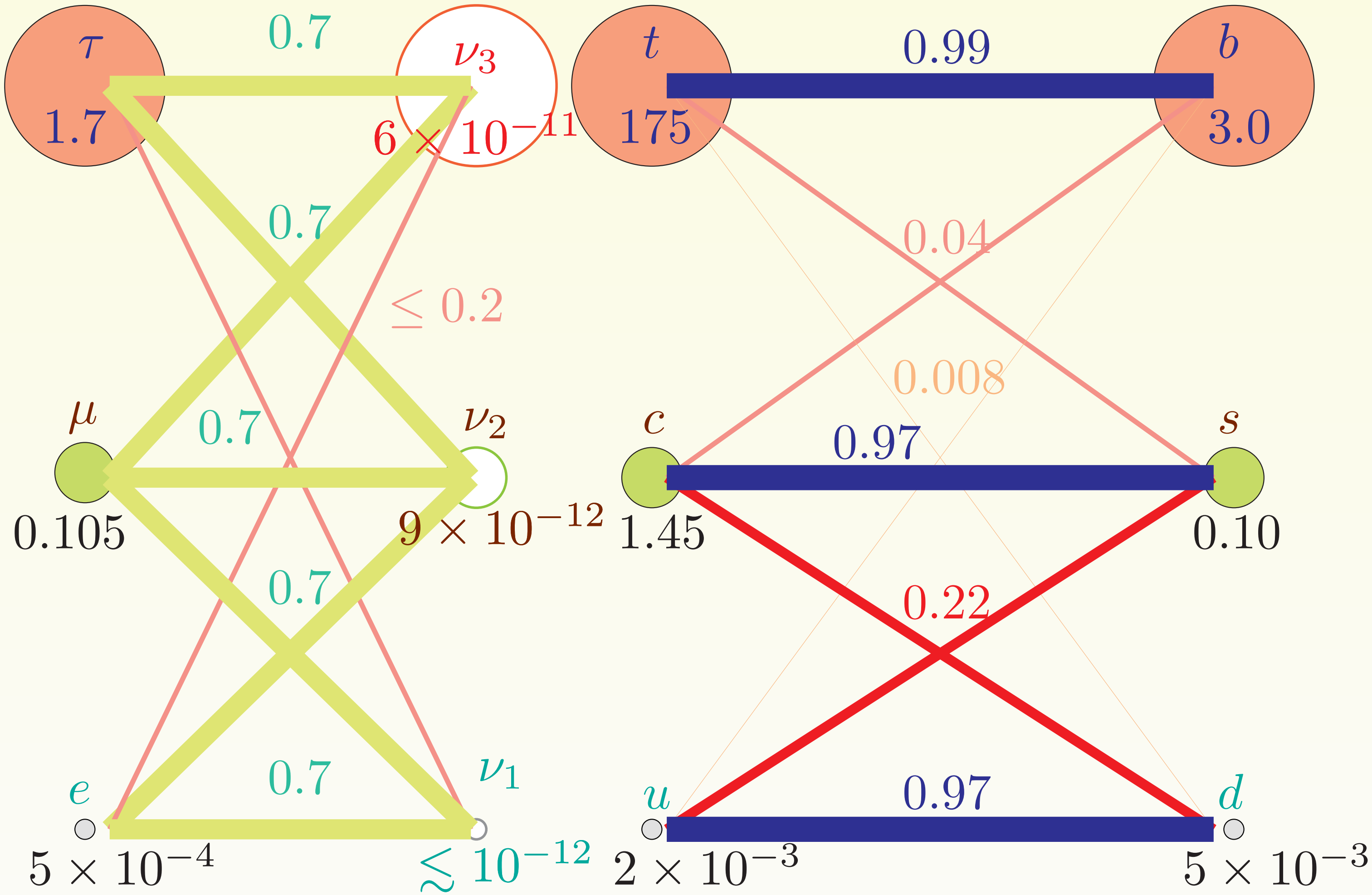
Flavour physics: who ordered that??

- 3 families with identical gauge quantum numbers.
- Strong hierarchy between generations.
- Small quark, large lepton mixing angles.



Flavour physics: who ordered that??

- 3 families with identical gauge quantum numbers.
- Strong hierarchy between generations.
- Small quark, large lepton mixing angles.



Standard Model

All flavour physics originate in Yukawa couplings:

$$\mathcal{L}_Y \;=\; H\,\bar{Q}_i\,Y^d_{ij}\,d_j \;+\; H^*\,\bar{Q}_i\,Y^u_{ij}\,u_j$$

In absence of Yukawas, $\mathcal{L}_{SM}$ invariant under global $(U(3))^5$

$\Longrightarrow$ quark masses and CKM mixings only observables in SM

Not enough information to determine the full Yukawa matrices

Supersymmetry

New flavour dependent interactions (sfermions/gauginos)

$\Longrightarrow$ new experimental information on flavour (urgently needed)!!

Flavour and CP problems

Flavour and CP problems

SUSY Flavour and CP

Soft masses fixed by $m_{3/2}$. $O(m_{3/2})$ elements in soft matrices.

$\Rightarrow$ **Severe FCNC problem !!!**

CP broken, we can expect all complex paramaters have $O(1)$ phases.

$\Rightarrow$ **Too large EDMs !!!**

Flavour and CP problems

SUSY Flavour and CP

Soft masses fixed by $m_{3/2}$. $O(m_{3/2})$ elements in soft matrices.

$\Rightarrow$ **Severe FCNC problem !!!**

CP broken, we can expect all complex paramaters have $O(1)$ phases.

$\Rightarrow$ **Too large EDMs !!!**

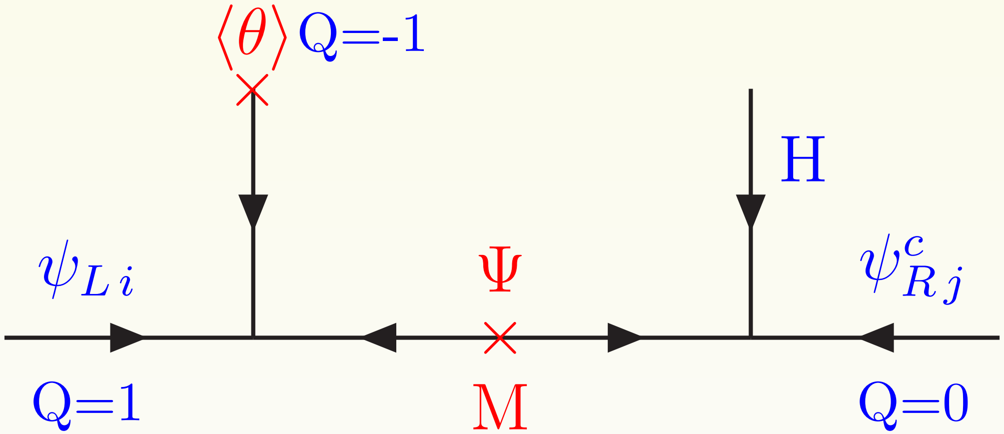
SM Flavour and CP

Fermion masses fixed by M_W. If $O(1)$ elements in Yukawa matrices

$\Rightarrow$ **Impossible reproduce masses, mixings !!!**

Flavour symmetries in SUSY

- Very different elements in Yukawa matrices: $y_t \simeq 1$, $y_u \simeq 10^{-5}$
- Expect couplings in a "fundamental" theory $\mathcal{O}(1)$
- Small couplings generated at higher order or function of small vevs.
- Froggatt-Nielsen mechanism and flavour symmetry to understand small Yukawa elements. Example: $U(1)_{fl}$



$$\Rightarrow Y_{ij} = \left(\frac{\langle\theta\rangle}{M}\right) \ll 1$$

- Flavour symmetry to understand masses and mixings in Yukawas.
- Yukawas forbidden by symmetry, generated only after Spontaneous Symmetry Breaking.

- Flavour symmetry to understand masses and mixings in Yukawas.
- Yukawas forbidden by symmetry, generated only after Spontaneous Symmetry Breaking.

- Unbroken symmetry applies both to fermion and sfermions.
- Diagonal soft masses allowed by symmetry.
- Nonuniversality in soft terms proportional to symm. breaking.

- Flavour symmetry to understand masses and mixings in Yukawas.
- Yukawas forbidden by symmetry, generated only after Spontaneous Symmetry Breaking.

- Unbroken symmetry applies both to fermion and sfermions.
- Diagonal soft masses allowed by symmetry.
- Nonuniversality in soft terms proportional to symm. breaking.

$$\Downarrow$$

We can relate the structure in Yukawa matrices to the nonuniversality in Soft Breaking masses !!!

Yukawa textures

- Masses and mixings in terms of a few fundamental parameters.
- Small mixing due to smallness of offdiagonal vs diagonal entries.
- Approximate texture zeros (GST) $\Rightarrow$ relate masses and mixings

Phenomenological fits:

$$Y_d \propto \begin{pmatrix} \leq \bar{\varepsilon}^5 & a\ \bar{\varepsilon}^3 & b\ \bar{\varepsilon}^3 \\ a\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & c\ \bar{\varepsilon}^2 \\ \leq \bar{\varepsilon} & \leq 1 & 1 \end{pmatrix}, \qquad Y_u \propto \begin{pmatrix} \leq \varepsilon^4 & \varepsilon^3 & \mathcal{O}(\varepsilon^3) \\ \leq \varepsilon^3 & \varepsilon^2 & \mathcal{O}(\varepsilon^2) \\ \leq \varepsilon & \leq 1 & 1 \end{pmatrix}$$

with $\varepsilon \simeq 0.05$ and $\bar{\varepsilon} \simeq 0.15$

Symmetric texture

- Non-Abelian flavour symmetries.

$$Y^d = \begin{pmatrix} 0 & 1.5\ \varepsilon^3 & 0.4\ \varepsilon^3 \\ 1.5\ \varepsilon^3 & \varepsilon^2 & 1.3\ \varepsilon^2 \\ 0.4\ \varepsilon^3 & 1.3\ \varepsilon^2 & 1 \end{pmatrix} y_b$$

- Universal sfermion masses in in unbroken limit:

$$\mathcal{L}_{m^2} = m_0^2 \Phi^\dagger \Phi = m_0^2 \left(\phi_1\ \phi_2\ \phi_3\right)^* \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}$$

- After symmetry breaking:

$$M^2_{\tilde{D}_R} \simeq \begin{pmatrix} 1+\bar{\varepsilon}^3 & \bar{\varepsilon}^3 & 0 \\ \bar{\varepsilon}^3 & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ 0 & \bar{\varepsilon}^2 & 1+\bar{\varepsilon} \end{pmatrix} m_0^2$$

Asymmetric texture

- Abelian flavour symmetries.

$$Y^d = \begin{pmatrix} \varepsilon^4 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon & 1 & 1 \end{pmatrix} y_b$$

- In principle nonuniversal masses in unbroken symmetry:

$$\mathcal{L}_{m^2} = m_1^2\ \phi_1^*\phi_1 + m_2^2\ \phi_2^*\phi_2 + m_3^2\ \phi_3^*\phi_3$$

- After symmetry breaking:

$$M^2_{\tilde{D}_R} \simeq \begin{pmatrix} 1 & \bar{\varepsilon} & \bar{\varepsilon} \\ \bar{\varepsilon} & c & b \\ \bar{\varepsilon} & b & a \end{pmatrix} m_0^2$$

$SU(3)$ Flavour model

- $Q, L \sim \mathbf{3}$ and $d^c, u^c, e^c \sim \mathbf{3}$; flavon fields: $\theta_3, \theta_{23} \sim \overline{\mathbf{3}}$, $\overline{\theta}_3, \overline{\theta}_{23} \sim \mathbf{3}$
- Family Symmetry breaking: $SU(3) \xrightarrow{\langle\theta_3\rangle} SU(2) \xrightarrow{\langle\theta_{23}\rangle} \emptyset$

$$\theta_3, \overline{\theta}_3 = \begin{pmatrix} 0 \\ 0 \\ a_3 \end{pmatrix}, \quad \theta_{23}, \overline{\theta}_{23} = \begin{pmatrix} 0 \\ b \\ b \end{pmatrix} \text{with } \left(\frac{a_3}{M}\right) \sim \mathcal{O}(1), \left(\frac{b}{M_u}\right) \simeq \left(\frac{b}{M_d}\right)^2 = \varepsilon \sim 0.05.$$

- Yukawa superpotential: $W_Y = H\psi_i\psi^c_j\left[\theta^i_3\theta^j_3 + \theta^i_{23}\theta^j_{23}\left(\theta_3\overline{\theta_3}\right) + \epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta^j_{23}\left(\theta_{23}\overline{\theta_3}\right)\right]$

$$Y^f = \begin{pmatrix} 0 & a\ \varepsilon^3 & b\ \varepsilon^3 \\ a\ \varepsilon^3 & \varepsilon^2 & c\ \varepsilon^2 \\ b\ \varepsilon^3 & c\ \varepsilon^2 & 1 \end{pmatrix} \frac{|a_3|^2}{M^2},$$

- Soft mass coupling $\Phi^\dagger\Phi$ invariant $\Longrightarrow$ common soft mass for the triplet
- Universality guaranteed in the exact symmetry limit.
- After symmetry breaking offdiagonal entries proportional to (complex) flavon vevs

$$M^2_{ij} = m_0^2 \Big(\delta^{ij} + \frac{1}{M^2}[\theta_3^{i\dagger}\theta_3^j + \overline{\theta}_3^{i\dagger}\overline{\theta}_3^j + \theta_{23}^{i\dagger}\theta_{23}^j + \overline{\theta}_{23}^{i\dagger}\overline{\theta}_{23}^j] + \frac{1}{M^4}[(\epsilon^{ikl}\overline{\theta}_{3,k}\overline{\theta}_{23,l})^\dagger(\epsilon^{jmn}\overline{\theta}_{3,m}\overline{\theta}_{23,n}) + (\epsilon_{ikl}\theta_3^k\theta_{23}^l)^\dagger(\epsilon_{jmn}\theta_3^m\theta_{23}^n)] + \ldots\Big)$$

$$M^2_{\tilde{D}_R}{}^{\mathrm{SCKM}} \simeq 6\, M^2_{1/2}\, \mathbb{1} + \begin{pmatrix} 1+\bar\varepsilon^3 & \bar\varepsilon^3 & \bar\varepsilon^3 \\ \bar\varepsilon^3 & 1+\bar\varepsilon^2 & \bar\varepsilon^2 \\ \bar\varepsilon^3 & \bar\varepsilon^2 & 1+\bar\varepsilon \end{pmatrix} m_0^2$$

(with $\bar\varepsilon \simeq 0.15$,$\varepsilon \simeq 0.05$)

- Soft mass coupling $\Phi^\dagger\Phi$ invariant $\Longrightarrow$ common soft mass for the triplet
- Universality guaranteed in the exact symmetry limit.
- After symmetry breaking offdiagonal entries proportional to (complex) flavon vevs

$$M^2_{ij} \;=\; m_0^2 \left(\delta^{ij} \;+\; \frac{1}{M^2}[\theta_3^{i\dagger}\theta_3^j \;+\; \overline{\theta}_3^{i\dagger}\overline{\theta}_3^j \;+\; \theta_{23}^{i\dagger}\theta_{23}^j \;+\; \overline{\theta}_{23}^{i\dagger}\overline{\theta}_{23}^j] \;+\; \frac{1}{M^4}[(\epsilon^{ikl}\overline{\theta}_{3,k}\overline{\theta}_{23,l})^\dagger(\epsilon^{jmn}\overline{\theta}_{3,m}\overline{\theta}_{23,n}) + (\epsilon_{ikl}\theta_3^k\theta_{23}^l)^\dagger(\epsilon_{jmn}\theta_3^m\theta_{23}^n)] + \ldots\right)$$

$$M^2_{\tilde{D}_R}{}^{\mathrm{SCKM}} \simeq 6\; M^2_{1/2}\;\mathbb{1} \;+\; \begin{pmatrix} 1 & 0.003 & 0.003 \\ 0.003 & 1 & 0.02 \\ 0.003 & 0.02 & 1 \end{pmatrix} m_0^2$$

(with $\bar{\varepsilon} \simeq 0.15$,$\varepsilon \simeq 0.05$)

At M_W in the SCKM basis:

$$M^2_{\tilde{D}_L} \simeq 6\ M^2_{1/2}\ \mathbb{1} + \begin{pmatrix} 1+\varepsilon^3 & \varepsilon^2\bar{\varepsilon} & \varepsilon^2\bar{\varepsilon} + c_{\rm run}\ \bar{\varepsilon}^3 \\ \varepsilon^2\bar{\varepsilon} & 1+\varepsilon^2 & \varepsilon^2 + c_{\rm run}\ \bar{\varepsilon}^2 \\ \varepsilon^2\bar{\varepsilon} + c_{\rm run}\ \bar{\varepsilon}^3 & \varepsilon^2 + c_{\rm run}\ \bar{\varepsilon}^2 & 1+\bar{\varepsilon} \end{pmatrix} m_0^2$$

$$M^2_{\tilde{E}_R} \simeq 0.15\ M^2_{1/2}\ \mathbb{1} + \begin{pmatrix} 1+\bar{\varepsilon}^3 & \frac{\bar{\varepsilon}^3}{3} & \bar{\varepsilon}^3 \\ \frac{\bar{\varepsilon}^3}{3} & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1+\bar{\varepsilon} \end{pmatrix} m_0^2$$

$$M^2_{\tilde{E}_L} \simeq 0.5\ M^2_{1/2}\ \mathbb{1} + \begin{pmatrix} 1+\varepsilon^3 & \frac{\varepsilon^2\bar{\varepsilon}}{3} & \varepsilon^2\bar{\varepsilon} + c_{\rm run}\ \bar{\varepsilon}^3 \\ \frac{\varepsilon^2\bar{\varepsilon}}{3} & 1+\varepsilon^2 & \varepsilon^2 + 3\ c_{\rm run}\ \bar{\varepsilon}^2 \\ \varepsilon^2\bar{\varepsilon} + c_{\rm run}\ \bar{\varepsilon}^3 & \varepsilon^2 + 3\ c_{\rm run}\ \bar{\varepsilon}^2 & 1+\varepsilon \end{pmatrix} m_0^2$$

At M_W in the SCKM basis:

$$M^2_{\tilde{D}_L} \simeq 6\, M^2_{1/2}\, \mathbb{1} + \begin{pmatrix} 1 & 4\times 10^{-4} & 7\times 10^{-4} \\ 4\times 10^{-4} & 1 & 4\times 10^{-3} \\ 7\times 10^{-4} & 4\times 10^{-3} & 1 \end{pmatrix} m_0^2$$

$$M^2_{\tilde{E}_R} \simeq 0.15\, M^2_{1/2}\, \mathbb{1} + \begin{pmatrix} 1 & 0.001 & 0.003 \\ 0.001 & 1 & 0.02 \\ 0.003 & 0.02 & 1 \end{pmatrix} m_0^2$$

$$M^2_{\tilde{E}_L} \simeq 0.5\, M^2_{1/2}\, \mathbb{1} + \begin{pmatrix} 1 & 1\times 10^{-4} & 7\times 10^{-4} \\ 1\times 10^{-4} & 1 & 1\times 10^{-2} \\ 7\times 10^{-4} & 1\times 10^{-2} & 1 \end{pmatrix} m_0^2$$

FCNC constraints

- Large offdiagonal entries in sfermion mass matrices generally overproduce FCNC and CP Violation transitions

$\Longrightarrow$ SUSY flavour problem

- Strong phenomenological bounds on Mass Insertions

$$\left(\delta^f_A\right)_{ij} = \frac{(m^2_{\tilde{f}_A})_{ij}}{m^2_{\tilde{f}}}$$

- Very stringent bounds on $d \to s$ transitions from ΔM_k and ε_k:

$$\mathrm{Re}\{\left(\delta^d_R\right)_{12}\} \leq 4 \times 10^{-2}, \qquad \mathrm{Im}\{\left(\delta^d_R\right)_{12}\} \leq 3.2 \times 10^{-3}$$

- Less stringent bounds from $b \to d$ and $b \to s$ transitions

$$\mathrm{Re}\{\left(\delta^d_R\right)_{13}\},\ \mathrm{Im}\{\left(\delta^d_R\right)_{13}\} \ \leq \ 0.1$$

($\Rightarrow$ Simple abelian models not allowed by ΔM_k and ε_k)

Spontaneous CP violation

- CP spontaneously broken in the flavour sector by complex flavon vevs.

$$\langle\theta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u & 0 \\ 0 & a_3^d\ e^{i\chi} \end{pmatrix}, \quad \langle\bar\theta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} a_3^u\ e^{i\alpha_u} & 0 \\ 0 & a_3^d\ e^{i\alpha_d} \end{pmatrix},$$

$$\langle\theta_{23}\rangle = \begin{pmatrix} 0 \\ b_{23} \\ b_{23}\ e^{i\beta_3} \end{pmatrix}, \quad \langle\bar\theta_{23}\rangle = \begin{pmatrix} 0 \\ b_{23}\ e^{i\beta_2'} \\ b_{23}\ e^{i(\beta_2'-\beta_3)} \end{pmatrix}.$$

- Model dependent: Explicit example from Ross, Velasco-Sevilla and O.V.

$$M^2_{\tilde{E}_R}(M^2_{\tilde{E}_L}) \propto \begin{pmatrix} 1+\bar\varepsilon^3 & \frac{\bar\varepsilon^3}{3} & \bar\varepsilon^3 e^{-i(\beta_3-\chi)} \\ \frac{\bar\varepsilon^3}{3} & 1+\bar\varepsilon^2 & \bar\varepsilon^2 e^{-i(\beta_3-\chi)} \\ \bar\varepsilon^3 e^{-i(\beta_3-\chi)} & \bar\varepsilon^2 e^{-i(\beta_3-\chi)} & 1+\bar\varepsilon \end{pmatrix} m_0^2$$

Solution to the SUSY CP problem

- $SU(3)_{fl}$ and CP spontaneously broken in Yukawas at $M \ll M_{\rm Planck}$
- At $M_{\rm Planck}$, Kähler (soft masses) real and universal and Guidice-Masiero μ term real.
- After $SU(3)$ breaking, Yukawa matrices and offdiagonal elements in soft masses contain $\mathcal{O}(1)$ CP violating phases (δ_{CKM})
- Trilinear couplings, Y^A, same (leading order) structure as Y.

$\Longrightarrow$ Diagonal elements in Y^A are real at leading order in SCKM basis.

Still contributions to EDMs from offdiagonal elements in sfermion masses:

$$d_e \propto \ (\delta^e_{LL})_{1i}(\delta^e_{LR})_{i1}\, f_1 + (\delta^e_{LR})_{1i}(\delta^e_{RR})_{i1}\, f_2 + (\delta^e_{LL})_{1i}(\delta^e_{LR})_{ij}(\delta^e_{RR})_{j1} f_3$$

WARNING!!!

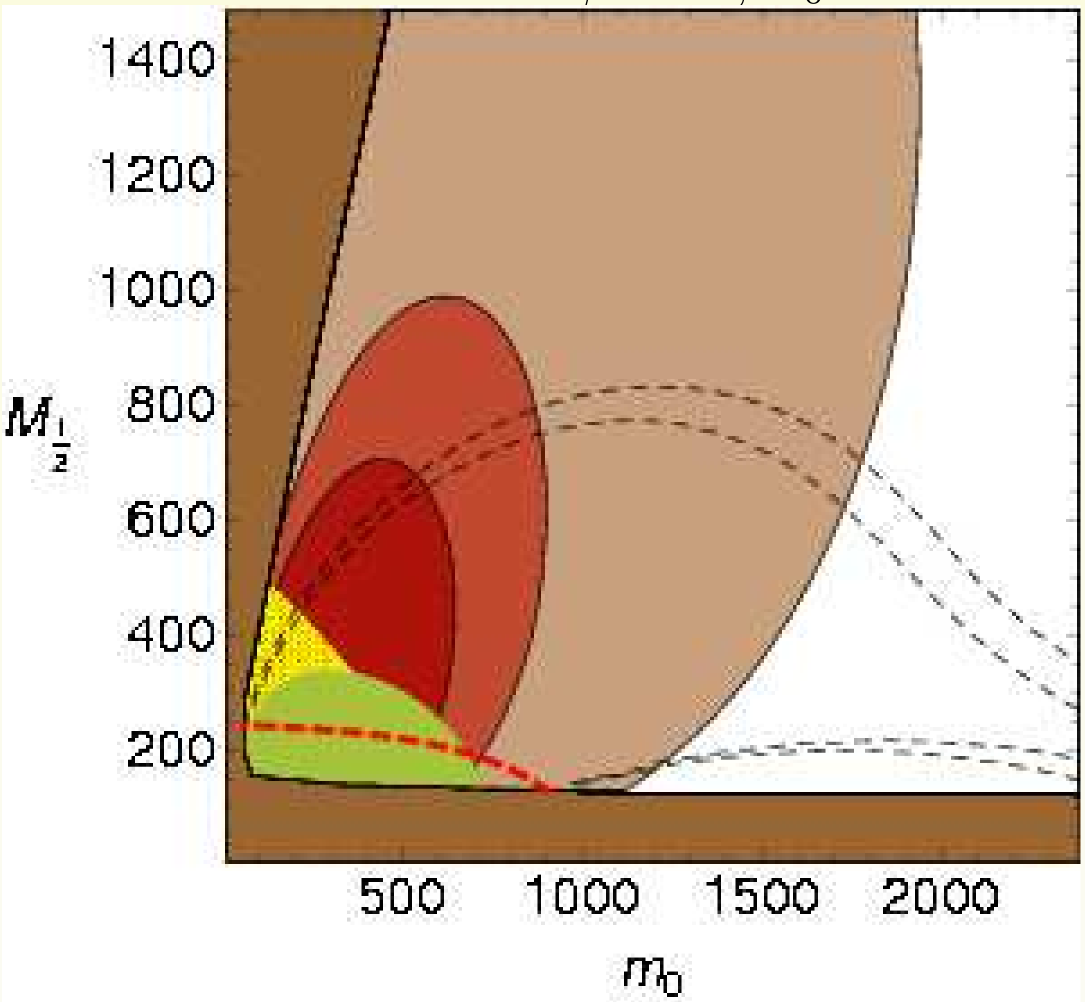
$O(1)$ coefficients in soft breaking terms completelly unknown.

Following plots are only true in order of magnitude !!

Predictions may easily vary by factors of 2, 3 . . .

lepton EDMs

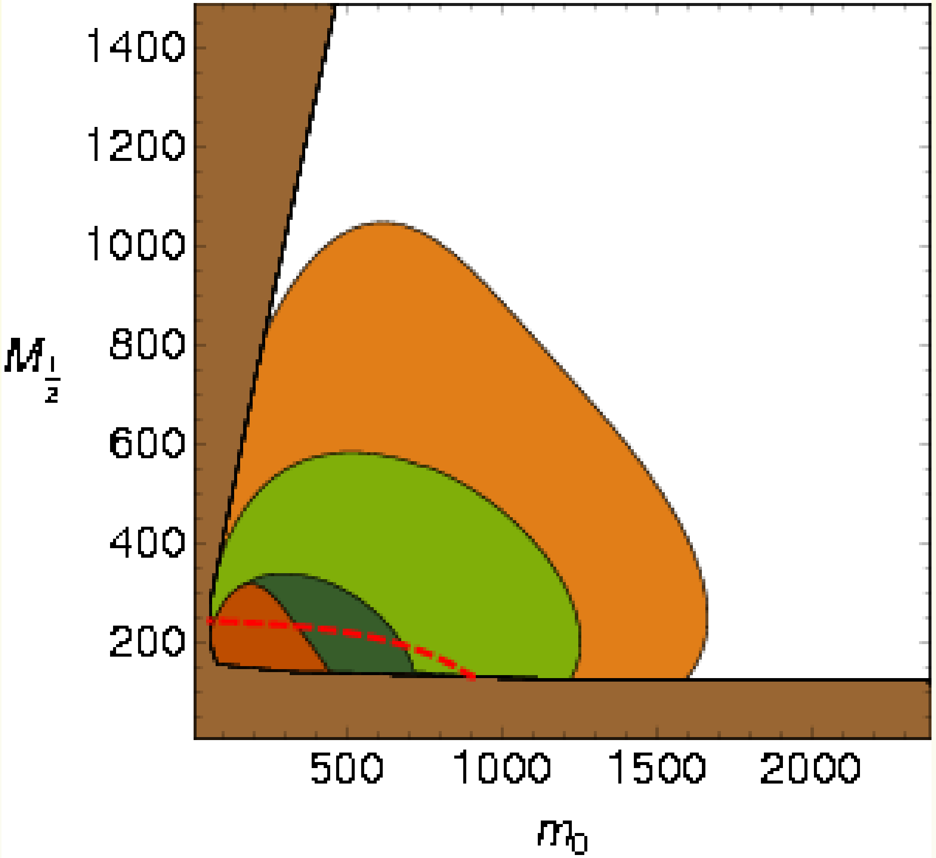
RVV model 1 $\tan\beta = 10,\ A_0 = 0$



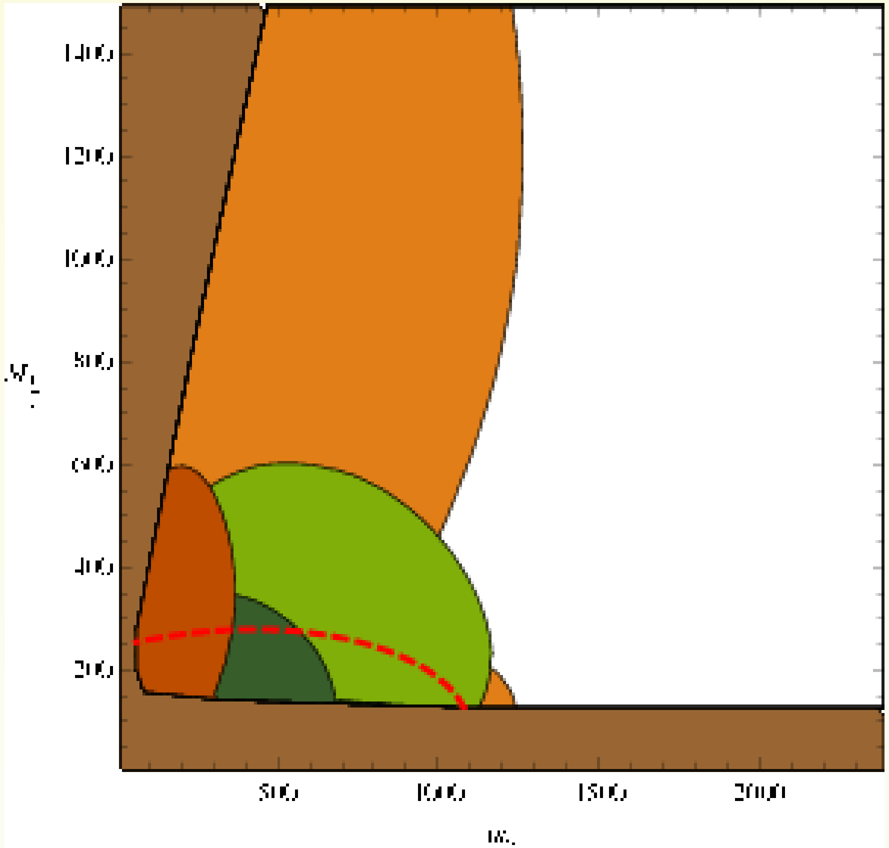
Dark red: $d_e = 10^{-29}$ e cm, Light red: $d_e = 5 \times 10^{-30}$ e cm, Grey: $d_e = 10^{-30}$ e cm
Green: LFV constraints, Yellow $(g-2)$ favoured region.

Lepton Flavour Violation

$\tan\beta = 10,\ A_0 = 0$



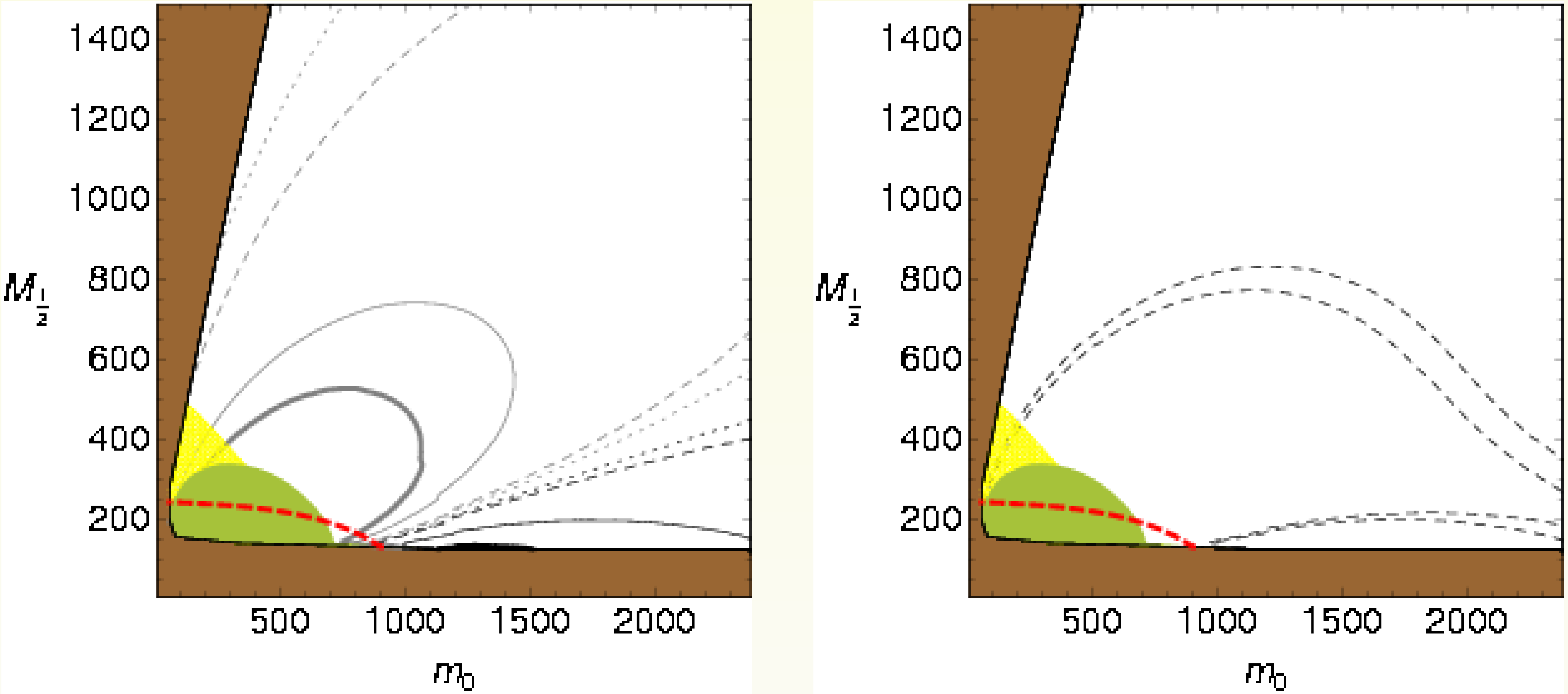
$\tan\beta = 10,\ A_0 = m_0$



Dark (light) Brown: Present (fut. 10^{-13}) $\mu \to e\gamma$ bound,
Dark (light) Green: Present (fut. 10^{-9}) $\tau \to \mu\gamma$ bound.

ε_K and B_s mixing

Large SUSY contributions to ε_K ($\tan\beta = 10,\ A_0 = 0$):



$\varepsilon_K^{\rm SUSY} = 10^{-3}, 5\times 10^{-3}, 10^{-4}$ $\qquad$ $\varepsilon_K^{\rm SUSY} + \varepsilon_K^{\rm SM} = \varepsilon_K^{\rm exp}$

However, things are difficult in B system...

- SM phase in B_s small: $\beta_S = 0.035$, where the SM contribution to mixing:

$$M_{12}^{\rm SM} \simeq \frac{\alpha_{em}^2}{8 M_W^2 \sin^2\theta_W} \, \frac{m_t^2}{M_W^2} \, \tfrac{1}{3} f_B^2 B_B \, \left(V_{tb}^* V_{ts}\right)^2$$

while SUSY contribution:

$$M_{12}^{\rm SUSY} \simeq \frac{\alpha_s^2}{216 M_{\rm SUSY}^2} \, f(x) \, \tfrac{1}{3} f_B^2 B_B \, \left(\delta_{LL}^d\right)_{12}^2$$

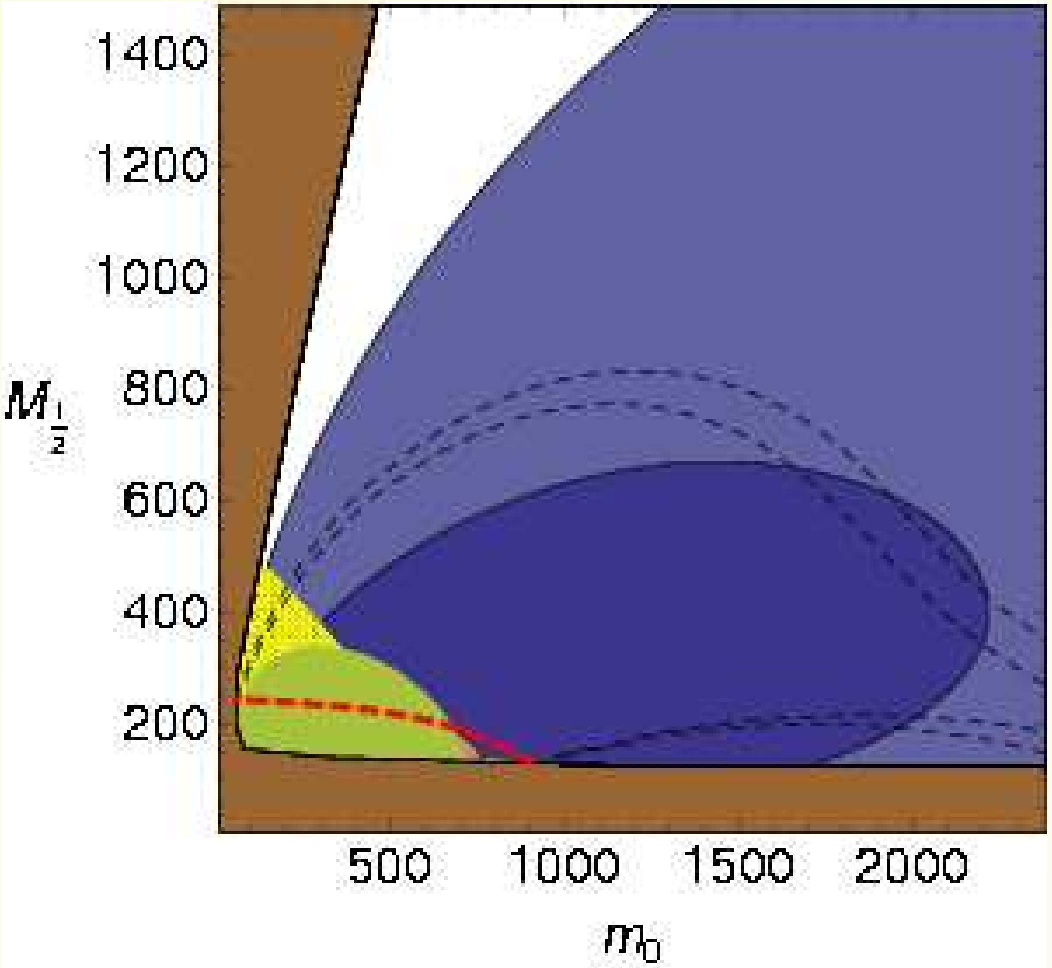
- To have a large phase in mixing $M_{12}^{B_s} = M_{12}^{\rm SM} + M_{12}^{\rm SUSY}$, we need,

$$1 \simeq \frac{M_{12}^{\rm SUSY}}{M_{12}^{\rm SM}} = \frac{\alpha_s^2 \sin^2\theta_W}{\alpha_{em}^2} \frac{M_W^2}{m_t^2 M_{\rm SUSY}^2} \frac{8 f(x)}{216} \frac{\left(\delta_{LL}^d\right)_{12}^2}{\left(V_{tb}^* V_{ts}\right)^2} = 12.5 \times 0.005 \times 0.04 \times \frac{\left(\delta_{LL}^d\right)_{12}^2}{(0.008)^2}$$

Thus, to have a large phase in B_s $\Rightarrow$ $\left(\delta_{LL}^d\right)_{12} \geq 0.16$

Also, $|M_{12}^{B_s}|$ roughly agrees with SM prediction ...

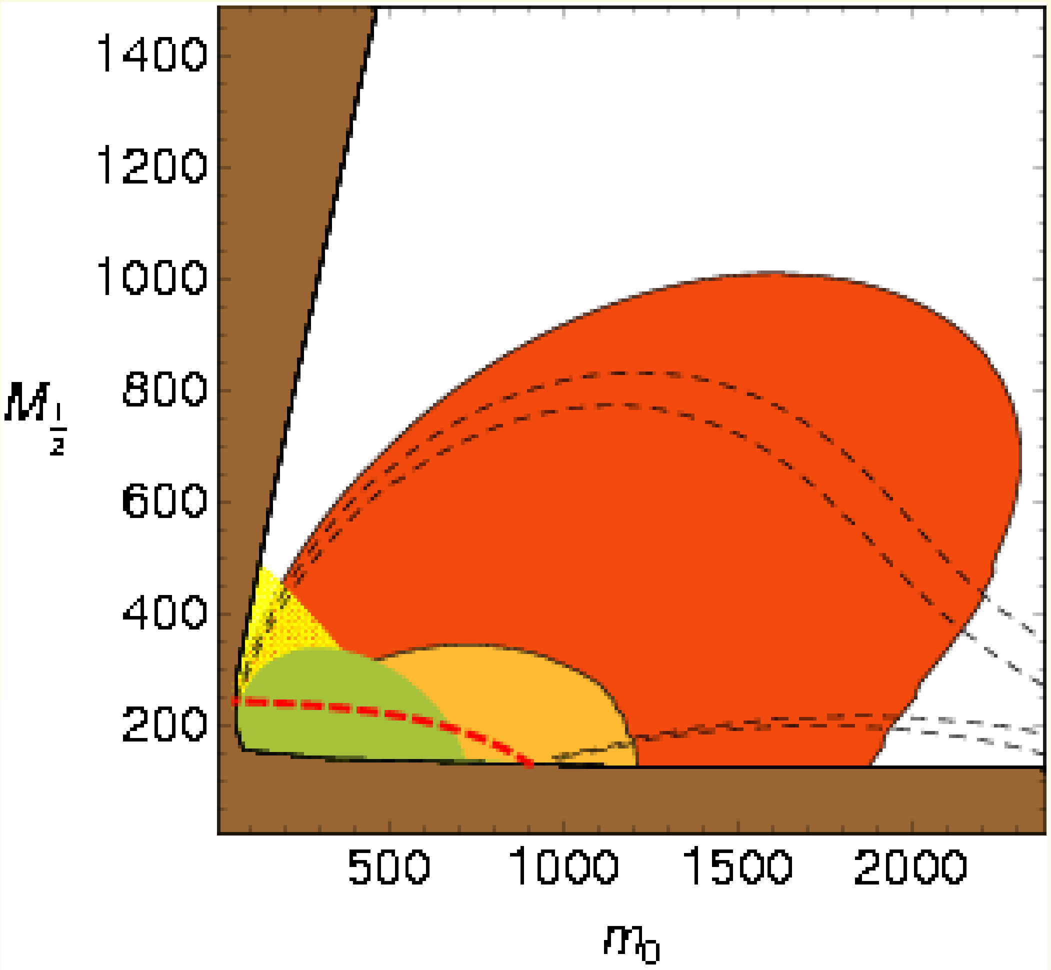
$$\phi_{B_s} = \arg\left[(M_{12}^{\rm SUSY} + M_{12}^{\rm SM})/M_{12}^{\rm SM}\right]$$



Dark blue: $\phi_{B_s} = 10^{-4}$, Light blue: $\phi_{B_s} = 10^{-5}$ with $\beta_s^{\rm SM} = 0.034$.

Neutron EDM

Chiral quark model, $\tan\beta = 10$



Orange: $d_n = 1 \times 10^{-28}$ e cm (Future bound), Red: $d_n = 1 \times 10^{-28}$ e cm

Conclusions

Flavour symmetries solve the Flavour and CP problems both in SUSY and in the SM!

$\Downarrow$

- Flavour phases (already obs. in Yukawas) contribute to EDMs.
- d_e and d_n in reach of proposed experiments for LHC sfermions.
- LFV processes ($\mu \to e\gamma$) close to present exp. limits.
- Sizeable contribution in the Kaon sector natural.
- LFV and EDMs can explore large areas of flavour MSSM in near future.

Field	θ_3	θ_{23}	$\overline{\theta_3}$	$\overline{\theta_{23}}$	Σ	H	ψ	W
SU(3)	$\bar{3}$	$\bar{3}$	3	1	1	3	1	1
U(1)	1	0	0	2	1	0	-1	0
$\mathbf{Z_9}$	0	0	1	-1	-2	0	0	0
$\mathbf{Z_6}$	0	1	0	-2	-2	0	0	0

with $\theta_3, \overline{\theta_3} \sim \mathbf{3} \oplus \mathbf{1}$, under $SU(2)_R$.

Yukawa Superpotential

$$W_Y = H\psi_i\psi_j^c \left[\theta_3^i\theta_3^j + \theta_{23}^i\theta_{23}^j\Sigma + \left(\epsilon^{ikl}\overline{\theta}_{23,k}\overline{\theta}_{3,l}\theta_{23}^j \left(\theta_{23}\overline{\theta_3}\right) + (i \leftrightarrow j)\right)\right.$$
$$\left. + X\,\epsilon^{ijk}\overline{\theta}_{23,k}\left(\theta_{23}\overline{\theta_3}\right)^2 + X\,\epsilon^{ijk}\overline{\theta}_{3,k}\left(\theta_{23}\overline{\theta_3}\right)\left(\theta_{23}\overline{\theta_{23}}\right) + \ldots\right]$$

Spontaneous Symmetry Breaking

Field	P	S	$\overline{S}$	T	U	$\overline{U}$	V	Y	Z
SU(3)	1	1	1	1	1	1	1	1	1
$\mathbf{U(1)_{PQ}}$	-2	$-9/2$	$9/2$	0	7	-7	-2	-9	-2
$\mathbf{Z_{15}}$	3	1	3	3	-2	-7	8	2	9

Then the flavon Superpotential,

$$W = P\left(T^4 + S^3\bar{S}^3\right) + U\left((\theta_{23}\bar{\theta}_{23}) + S^2\right) + V\left((\theta_3\bar{\theta}_3)^4 + S\bar{S}\right) + \bar{U}\bar{S}^2(\theta_3\bar{\theta}_3)$$
$$+ Y(\theta_3\bar{\theta}_2) + Z\left((\theta_{23}\bar{\theta}_3)(\theta_{23}\bar{\theta}_2) + \bar{S}^2\right) + \mu H_1 H_2\left[1 + (\theta_{23}\bar{\theta}_3)^5 + \ldots\right]$$

Phase determination: 1) T gets a vev radiatively with $\varphi_T = \frac{n\pi}{5}$,

2) $\varphi_{(S\bar{S})} = \frac{n'4}{3}\,\varphi_T$, 3) $\alpha_3 = \frac{m}{4}\,\varphi_{(S\bar{S})}$, 4) $\beta_3 = \frac{m'\pi}{5} - \alpha_3$