Precision calculations in $\mathrm{Br}(\bar{B} \to X_s\gamma)$

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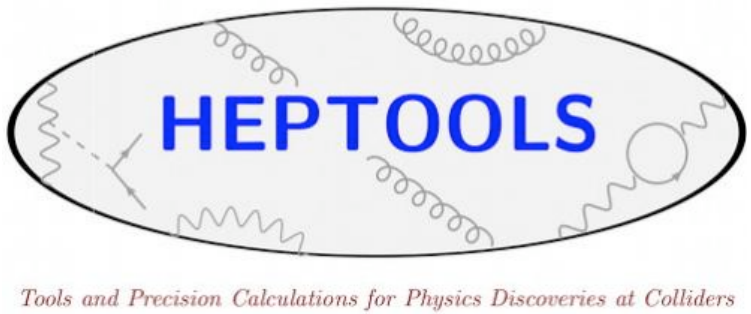
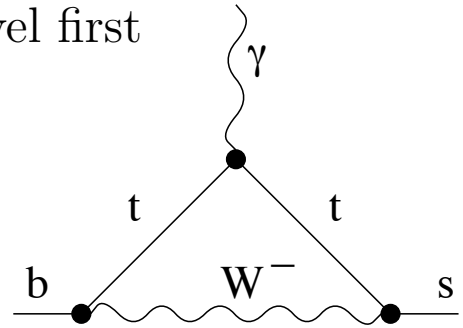


Motivation / differential decay rate

Sensitive to New Physics; in the SM generated at the 1-loop level first



→ constrain NP using experimental measurement

The differential decay rate is given by

$$d\Gamma\left(\bar{B}\to X_s\gamma\right)=\frac{G_F^2\alpha_{\rm em}m_b^2}{16\pi^3 m_B}\,|V_{tb}V_{ts}^*|^2\frac{d^3q}{(2\pi)^3 2E_\gamma}\sum_{ij}C_i^{\rm eff}C_j^{\rm eff}W^{ij}_{\mu\nu\alpha\beta}P^{\mu\nu\alpha\beta}$$

with the hadronic tensor

$$W^{ij}_{\mu\nu\alpha\beta}=\sum_X\int\frac{d^3p_X}{(2\pi)^3 2E_X}(2\pi)^4\delta^{(4)}(p_B-p_X-q)$$
$$\times\langle\bar{B}(p_B)|O_i^\dagger(0)|X_s(p_X)\rangle\langle X_s(p_X)|O_j(0)|\bar{B}(p_B)\rangle$$

and the photon tensor

$$P^{\mu\nu\alpha\beta}=\sum_{\lambda=\pm1}\langle 0|F^{\mu\nu}|\gamma(q,\lambda)\rangle\langle\gamma(q,\lambda)|F^{\alpha\beta}|0\rangle$$

Operator product expansion

The hadronic tensor is the imaginary part of the $\bar{B} \to \bar{B}$ scattering amplitude:

$$W^{ij}_{\mu\nu\alpha\beta} = 2\,\mathrm{Im}\left(i \int d^4x\, e^{-iq\cdot x} \langle \bar{B}(p_B)|\, T\Big\{ O_i^\dagger(x) O_j(0) \Big\}\, |\bar{B}(p_B)\rangle \right)$$

From $\Lambda_{\mathrm{QCD}} \approx m_B - m_b \ll m_b$ follows

$$W^{ij}_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta}\left(\bar{B} \to X_s\gamma\right) = W^{ij}_{\mu\nu\alpha\beta} P^{\mu\nu\alpha\beta}\left(b \to X_s^{\mathrm{partonic}}\gamma\right) \;+\; O\left(\frac{\Lambda_{\mathrm{QCD}}}{m_b}\right)$$

Systematic expansion in $\Lambda_{\mathrm{QCD}}/m_b$; leading term is the partonic decay rate:

$$d\Gamma\left(\bar{B} \to X_s^{\mathrm{partonic}}\gamma\right) = \frac{G_F^2 \alpha_{\mathrm{em}} m_b^2}{16\pi^3 m_B} |V_{tb}V_{ts}^*|^2 \frac{d^3q}{(2\pi)^3 2E_\gamma}$$

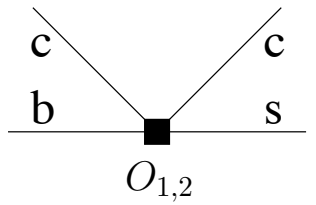
$$\times \sum_{ij} C_i^{\mathrm{eff}}(\mu) C_j^{\mathrm{eff}}(\mu) G_{ij}(E_\gamma, \mu)$$

Effective Lagrangian relevant for $b \to s\gamma$

The decay involves a large separation of scales ($m_b \ll M_W$) and is most conveniently described in the framework of an effective theory

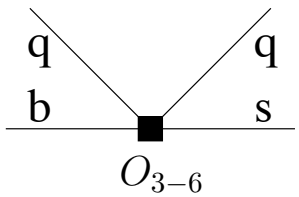
Effective Lagrangian resulting from Standard Model interactions:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{\text{light fermions}}_{\text{QCD}\times\text{QED}} + \frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i + \dots$$



$O_{1,2}$

At leading order in electroweak interactions:



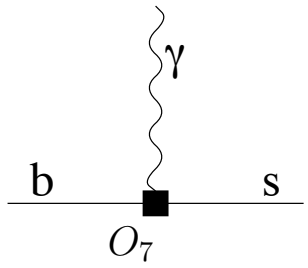
O_{3-6}

$$O_1 = (\bar{s}_L\gamma_\mu T^a c_L)(\bar{c}_L\gamma^\mu T^a b_L)\,, \quad O_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

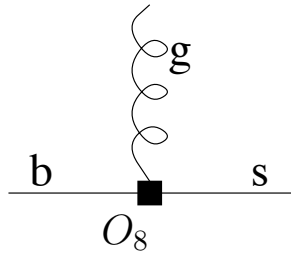
$$O_3 = (\bar{s}_L\gamma_\mu b_L)\sum\nolimits_q (\bar{q}\gamma^\mu q)\,, \quad O_4 = (\bar{s}_L\gamma_\mu T^a b_L)\sum\nolimits_q (\bar{q}\gamma^\mu T^a q)$$

$$O_5 = (\bar{s}_L\gamma_{\mu\nu\lambda} b_L)\sum\nolimits_q (\bar{q}\gamma^{\mu\nu\lambda} q)\,, \quad O_6 = (\bar{s}_L\gamma_{\mu\nu\lambda} T^a b_L)\sum\nolimits_q (\bar{q}\gamma^{\mu\nu\lambda} T^a q)$$

$$O_7 = \frac{em_b}{16\pi^2}\,(\bar{s}_L\sigma^{\mu\nu} b_R)F_{\mu\nu}\,, \quad O_8 = \frac{gm_b}{16\pi^2}\,(\bar{s}_L\sigma^{\mu\nu} T^a b_R)G^a_{\mu\nu}$$



O_7



O_8

Matching conditions at $O(\alpha_s^2)$

Evaluate the Wilson coefficients $C_i(\mu)$ at the high-energy scale $\mu_0 \sim M_W$ by matching the Green functions of the effective theory with those of the Standard Model

Neglecting QED effects, the Wilson coefficients have the following perturbative expansion:

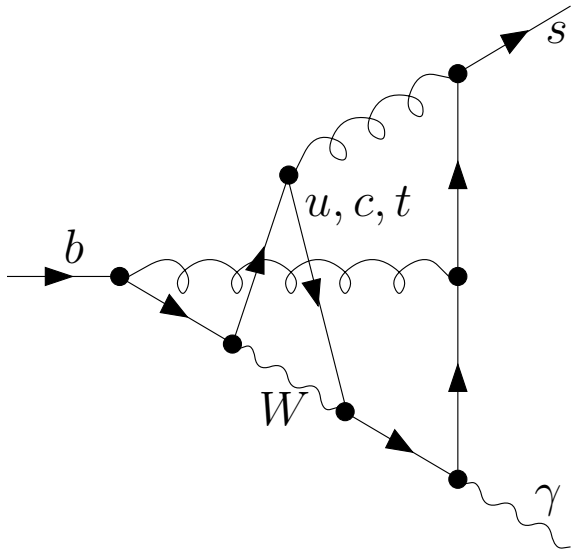
$$C_i(\mu) = C_i^{(0)}(\mu) + \alpha_s(\mu) C_i^{(1)}(\mu) + \alpha_s^2(\mu) C_i^{(2)}(\mu) + \ldots$$

One needs 3-loop calculation in the full theory to fix $C_7^{(2)}(\mu_0)$ & $C_8^{(2)}(\mu_0)$

Misiak, Steinhauser, hep-ph/0401041

For other operators $O(\alpha_s^2)$ means 2-loop;
done some time ago

→ Matching complete for $b \to s\gamma$ at $O(\alpha_s^2)$



Anomalous dimension matrix

Calculate the anomalous dimension matrix (γ_{ij}) which describes the mixing of operators,

$$\gamma_{ij} = Z_{ik}\,\mu\frac{d}{d\mu}(Z^{-1})_{kj}, \qquad C_i^{\rm bare} = Z_{ji}C_j$$

and solve the renormalization group equation (RGE) for the Wilson coefficients to evolve them from μ_0 down to the low-energy scale $\mu_b \sim m_b$,

$$\mu\frac{d}{d\mu}C_i = \gamma_{ji}C_j$$

$\longrightarrow$ Large logarithms are contained in $C_i(\mu_b)$

LO: $\alpha_s^n L^n$; NLO: $\alpha_s^n L^{n-1}$; NNLO: $\alpha_s^n L^{n-2}$; $L = \ln(m_b/M_W)$

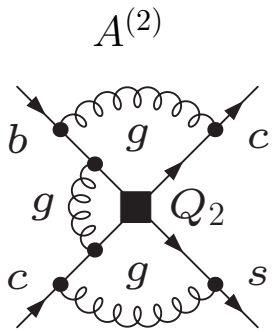
Anomalous dimension matrix (continued)

Neglecting QED effects, the ADM has the following perturbative expansion:

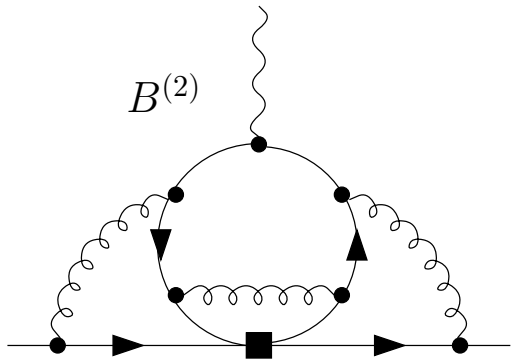
$$\gamma = \alpha_s \gamma^{(0)} + \alpha_s^2 \gamma^{(1)} + \alpha_s^3 \gamma^{(2)} + \ldots \qquad \gamma^{(n)} = \begin{pmatrix} A^{(n)}_{6\times 6} & B^{(n)}_{6\times 2} \\ 0_{2\times 6} & C^{(n)}_{2\times 2} \end{pmatrix}$$

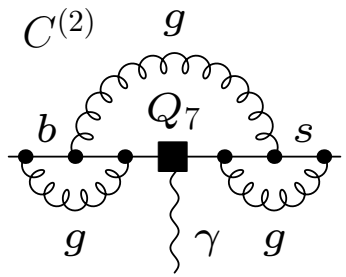
3-loop mixing in $(O_1, \ldots, O_6)$ & (O_7, O_8) sectors:
Gorbahn, Haisch, hep-ph/0411071
Gorbahn, Haisch, Misiak, hep-ph/0504194

4-loop mixing of $(O_1, \ldots, O_6)$ into O_7 and O_8:
Czakon, Haisch, Misiak, hep-ph/0612329



→ Wilson coefficients at $\mu \sim m_b$ up to $O(\alpha_s^2)$ known





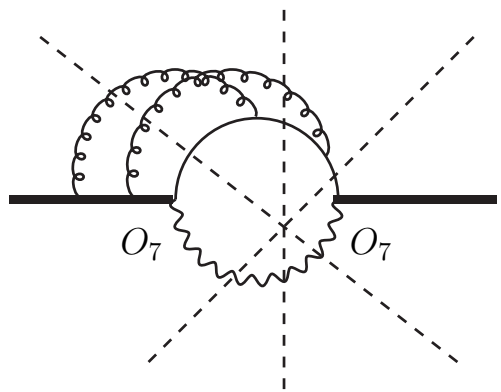
Low-energy matrix elements

Evaluate on-shell amplitudes of operators O_i at $\mu \sim m_b$

$$\longrightarrow \quad G_{ij}(E_\gamma, \mu) = Y_{ij}^{(0)} \delta_{i7}\delta_{j7} + \alpha_s(\mu) Y_{ij}^{(1)}(E_\gamma, \mu) + \alpha_s^2(\mu) Y_{ij}^{(2)}(E_\gamma, \mu) + \ldots$$

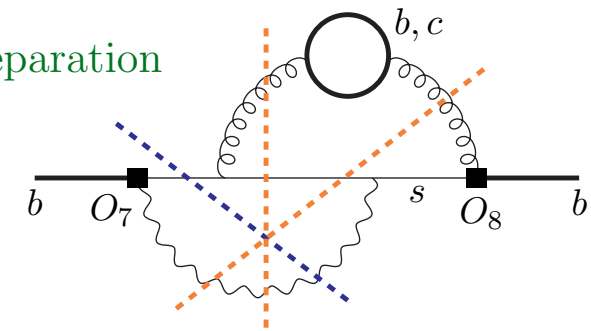
Calculation of (O_7, O_7)-interference is finished:

Blokland, Czarnecki, Misiak, Slusarczyk, Tkachov, hep-ph/0506055
Asatrian, TE, Greub, Hurth, Hovhannisyan,Poghosyan, hep-ph/0605009
Melnikov, Mitov, hep-ph/0505097
Asatrian,TE, Ferroglia, Gambino, Greub, hep-ph/0607316
Asatrian, TE, Gabrielyan, Greub, hep-ph/0611123



Calculation of (O_7, O_8)-interference still in progress:

Bieri, Greub, Steinhauser, hep-ph/0302051
Ligeti, Luke, Manohar, Wise, hep-ph/9903305
TE, arXiv:0805:3911
Asatrian, TE, Ferroglia, Greub, Ossola, in preparation



Some technical details of the calculation

At $O(\alpha_s^2)$ the following subprocesses are involved:

$$b \to s\gamma,\ b \to s\gamma g,\ b \to s\gamma gg,\ b \to s\gamma q\bar{q}$$

We calculate the individual cut contributions of 3-loop b-quark self-energies

We used Laporta algorithm to reduce the 2-particle-cuts to a set of MIs

This works also for the 3- & 4-particle-cuts after rewriting the PS into loop integrals

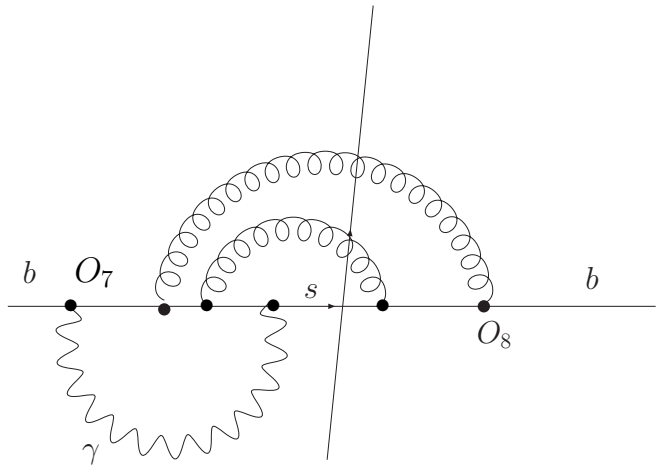
$$\delta(x) = \frac{1}{2\pi i}\left(\frac{1}{x - i\eta} - \frac{1}{x + i\eta}\right)$$

and introducing an additional constraint

$$\int_{E_0}^{E_{\max}} dE_\gamma\ \delta(E_\gamma - p_b \cdot p_\gamma / m_b)$$

to take the cut in the photon energy into account

Anastasiou *et al*, hep-ph/0312266



Calculation of master integrals

For massive quark bubble contributions we used Mellin-Barnes representation

$$(x+y)^{-\lambda} = \frac{1}{\Gamma(\lambda)} \int_C \frac{ds}{2\pi i} \frac{x^s}{y^{\lambda+s}} \Gamma(-s)\Gamma(\lambda+s)$$

and summed up all residue enclosed by the integration contour

For the remaining 2-particle-cuts we used sector decomposition together with suitable contour deformations

Nagy, Soper, hep-ph/0610028
Lazopoulos, Melnikov, hep-ph/0703273
Anastasiou, Beerli, Daleo, hep-ph/0703282

This part of the calculation is finished since 2007

The evaluation of the 3- & 4-particle-cuts is still in progress
Here we apply the method of differential equations

Remiddi, hep-th/9711188

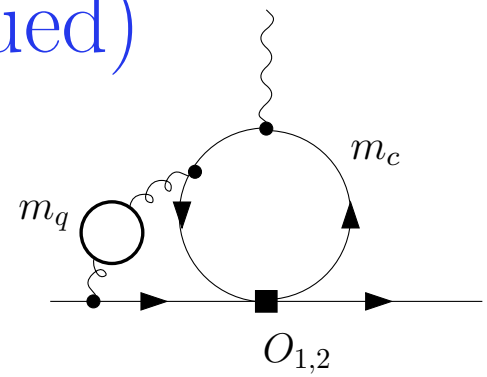
and numerical methods

Binoth, Heinrich, hep-ph/0004013; Anastasiou *et al*, hep-ph/0311311

Low-energy matrix elements (continued)

Calculation of (O_2, O_7)-interference still in progress:

Bieri, Greub, Steinhauser, hep-ph/0302051
Misiak, Steinhauser, hep-ph/0609241
Boughezal, Czakon, Schutzmeier, arXiv:0707:3090

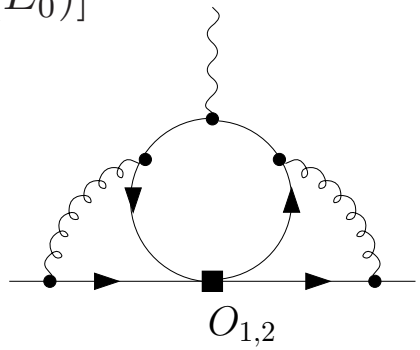


Interpolation in m_c is used to obtain the (O_2, O_7)-interference at the physical value of m_c

$$\mathcal{B}[\bar{B} \to X_s\gamma]_{E_\gamma > E_0} = \mathcal{B}[\bar{B} \to X_c e\bar{\nu}]_{\rm exp} \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi\, C} \left[P(E_0) + N(E_0)\right]$$

$$\frac{\Gamma[b \to X_s\gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2\, \Gamma[b \to X_u e\bar{\nu}]} = \left|\frac{V_{ts}^* V_{tb}}{V_{cb}}\right|^2 \frac{6\alpha_{\rm em}}{\pi}\, P(E_0).$$

$$C = \left|\frac{V_{ub}}{V_{cb}}\right|^2 \frac{\Gamma[\bar{B} \to X_c e\bar{\nu}]}{\Gamma[\bar{B} \to X_u e\bar{\nu}]}$$



Perturbative expansion of $P(E_0)$:

$$P(E_0) = P^{(0)} + \alpha_s^2 \left(P_1^{(1)} + P_2^{(1)}(E_0, r)\right) + \alpha_s^2 \left(P_1^{(2)} + P_2^{(2)}(E_0, r) + P_3^{(2)}(E_0, r)\right), \quad r = \frac{m_c}{m_b}$$

Calculated $P_2^{(2)}$ completely for $m_c \gg m_b/2$, $\qquad P_2^{(2)}(E_0, r) = \sum_{i,j} C_i^{(0)}(\mu_b) C_j^{(0)}(\mu_b) K_{ij}(E_0, r)$

$P_2^{(2)} = P_2^{(2)\beta_0} + P_2^{(2)\rm rem} \longrightarrow P_2^{(2)\rm rem}$ via interpolation in m_c

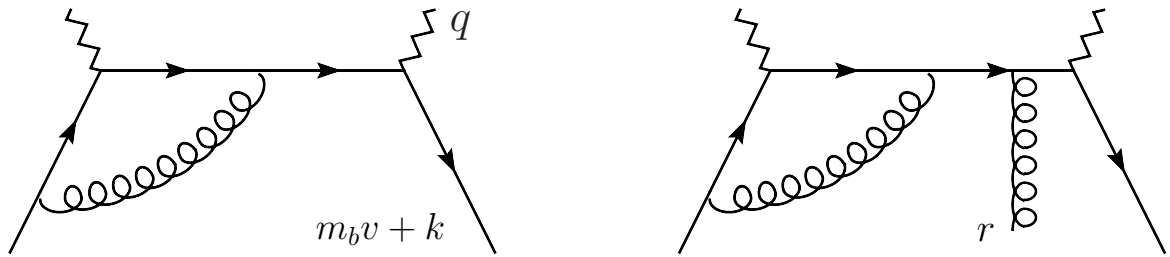
Nonperturbative powercorrections

Powercorrections start at $O(\Lambda_{\rm QCD}^2/m_b^2)$:

Falk, Luke, Savage, hep-ph/9308288
Bauer, hep-ph/9710513

$$W^{ij}_{\mu\nu\alpha\beta}P^{\mu\nu\alpha\beta}\left(\bar{B}\to X_s\gamma\right) = W^{ij}_{\mu\nu\alpha\beta}P^{\mu\nu\alpha\beta}\left(b\to X_s^{\rm partonic}\gamma\right)$$
$$+\frac{\kappa_1(E_\gamma)}{2m_b^2}\langle\bar{B}|\bar{b}_v(iD)^2b_v|\bar{B}\rangle$$
$$+\frac{\kappa_2(E_\gamma)}{2m_b^2}\langle\bar{B}|\bar{b}_v g_s\sigma^{\mu\nu}G_{\mu\nu}b_v|\bar{B}\rangle \;+\; O\left(\frac{\Lambda_{\rm QCD}^3}{m_b^3}\right)$$

At $O(\alpha_s\Lambda_{\rm QCD}^2/m_b^2)$ one has to calculate imaginary part of one-loop diagrams



The calculation of the full QCD diagrams is finished; right now we are working on the matching onto HQET TE, Gambino, Nandi, in preparation

Numerical results at NNLO

Standard Modell prediction at NNLO ($E_\gamma > 1.6\,\text{GeV}$):

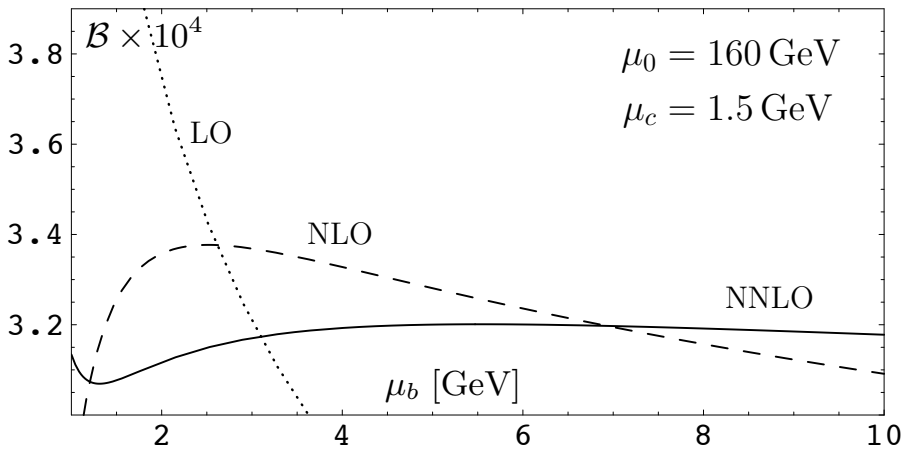
$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\text{SM}} = \begin{cases} (3.15 \pm 0.23) \times 10^{-4} & \text{Misiak et al, hep-ph/0609232} \quad \text{(1S scheme)} \\ (3.26 \pm 0.24) \times 10^{-4} & \text{(kinetic scheme)} \end{cases}$$

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\text{exp.}} = (3.52 \pm 0.23 \pm 0.09) \times 10^{-4} \qquad \text{HFAG, arXiv:0808.1297}$$

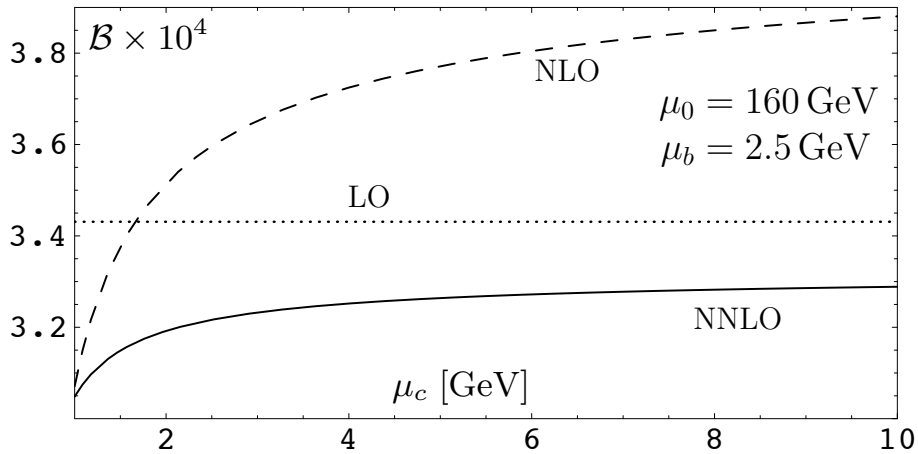
Contributions to total uncertainty:

3%	m_c-interpolation	3%	higher orders $O(\alpha_s^3)$
3%	parametric: $\alpha_s(M_Z)$, $\mathcal{B}^{\text{exp}}_{\text{semileptonic}}$, m_c & C, ...	5%	non-perturbative $O(\alpha_s\Lambda_{\text{QCD}}/m_b)$

Residual renormalization scale dependence:



low-energy scale



charm mass renormalization scale

Outlook / summary

Currently known but not yet included effects:

- 4-loop mixing of O_{1-6} into O_8 Czakon, Haisch, Misiak, hep-ph/0612329
- Some large-β_0 corrections to the spectrum at $O(\alpha_s^2)$
  Ligeti, Luke, Manohar, Wise, hep-ph/9903305; Ferroglia, Haisch, to be published
- Massive quark bubble effects in (O_7, O_7), $(O_{1,2}, O_7)$ and (O_7, O_8)
  Asatrian, TE, Gabrielyan, Greub, hep-ph/0611123
  Boughezal, Czakon, Schutzmeier, arXiv:0707.3090; TE, arXiv:0805:3911
- Non-perturbative $O(\alpha_s \Lambda_{\rm QCD}/m_b)$ effects $\sim C_7 C_8$ Lee, Neubert, Paz, hep-ph/0609224

Presently unknown contributions:

- Complete calculation of (O_7, O_8) and (O_2, O_7)-interference

To summarize:

- Theoretical uncertainty of the Standard Model prediction of $\mathcal{B}(\bar{B} \to X_s\gamma)$ at NNLO is on the level of the experimental uncertainty
- This allows for improved constraints on parameters of New Physics scenarios (2HDMs, UED models, ...)