Neutrinos and Lepton-Flavour–Violation

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Disclaimer:

- I am not THE Expert on Neutrinos and LFV.
- Comments and Corrections from the Audience are welcome and will not be taken personally . . .

Many Results and Inspirations taken from the Review in:

- M. Raidal *et al.*, “Flavour physics of leptons and dipole moments”, Eur. Phys. J. C **57** (2008) 13 [arXiv:0801.1826 [hep-ph]].

1. Neutrinos
   - Neutrino-Mixing Parameters from Experiment
   - Flavour Mixing in the Lepton Sector
   - Origin of Flavour Hierarchies?
2. LFV
   - Phenomenological Status of LFV
   - LFV Phenomenology in Specific Models
   - MFV in the Lepton Sector
3. Summary

Motivation

- Neutrino oscillations experimentally established. $\mapsto$
- Direct hint towards New Physics (GUTs).
  - See-saw scale $\sim$ GUT scale $\sim$ scale of $\not{L}$.
  - ν_R fits into 16-plet of $SO(10)$.
- Allows for Leptogenesis scenarios:
  - Out-of-equilibrium L-violating decays of heavy Majorana neutrinos.
  - L-asymmetry transformed into B-asymmetry via sphaleron processes.
  - Interference between tree-level and loop diagrams depends on Majorana phases.
  - For distinguishable flavours (i.e. τ from e, μ), also sensitive to light neutrino mixing matrix $U_{\rm PMNS}$.
  - For hierarchical heavy neutrinos, lightest mass should be in the range 10^7–10^9 GeV.

[Buchmüller et al. 02; Giudice et al. 03]

$\Rightarrow$ Physics close to the GUT scale.

- LFV violating effects tiny in minimally extended SM:

(e.g. $\mathcal{B}[\mu \to e\gamma]_{\rm SM} \sim 10^{-54}$] compared to $\mathcal{B}[\mu \to e\gamma]_{\rm exp.} < 10^{-11(13)}$)

$\Rightarrow$ High sensitivity to NP at the TeV scale.

parameter	best fit	2σ	3σ
Δm^2_{21} $[10^{-5}\ \mathrm{eV}^2]$	$7.65^{+0.23}_{-0.20}$	7.25–8.11	7.05–8.34
$\lvert\Delta m^2_{31}\rvert$ $[10^{-3}\ \mathrm{eV}^2]$	$2.40^{+0.12}_{-0.11}$	2.18–2.64	2.07–2.75
$\sin^2\theta_{12}$	$0.304^{+0.022}_{-0.016}$	0.27–0.35	0.25–0.37
$\sin^2\theta_{23}$	$0.50^{+0.07}_{-0.06}$	0.39–0.63	0.36–0.67
$\sin^2\theta_{13}$	$0.01^{+0.016}_{-0.011}$	$\lesssim 0.040$	$\lesssim 0.056$

Best-fit values for the three-flavour neutrino oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments.

[Schwetz/Tortola/Valle, arXiv:0808.2016]

- Two distinct mass^2 differences: $\lvert\Delta m^2_{31}\rvert \gg \Delta m^2_{21}$.
- Mixing angle θ_{23} close to maximal.
- Mixing angle θ_{12} large.
- Mixing angle θ_{13} small.

(Still) Open Issues in Neutrino Parameters

- Normal, inverted hierarchy, or almost democratic spectrum ?
- Absolute neutrino mass scale ?
- How small is θ_{13} ?
- Determination of CP-phases ?
- Majorana neutrinos ? (neutrino-less double β-decay)
- ...

1.2. Flavour Mixing in the Lepton Sector

- Yukawa sector in the (original) **SM**:

$$(Y_E)^{ij}\, (\bar{L}^i\, H)\, E_R^j \; + \; \text{h.c.}$$

  - massless (anti-) neutrinos with (positive) negative helicity
  - individual lepton flavour (L_e, L_μ, L_τ) conserved → No Mixing

- SM plus right-handed **Dirac Neutrinos** (i.e. L-conservation enforced):

$$(Y_E)^{ij}\, (\bar{L}^i\, H)\, E_R^j + (Y_\nu)^{ij}\, (\bar{L}^i\, \widetilde{H})\, \nu_R^j \; + \; \text{h.c.}$$

  - analogous to quark sector (CKM-like mixing)
  - phenomenologically and theoretically less interesting

- Minimally extended SM (as **Effective Theory**):

$$(Y_E)^{ij}\, (\bar{L}^i\, H)\, E_R^j + \frac{(g_\nu)^{ij}}{\Lambda_L}\, (\bar{L}^i \widetilde{H})(\widetilde{H}^\dagger L^j)^c + \text{h.c.}$$

  - effective dim-5 operator violates lepton number.
  - mismatch between diagonalization of Y_E and $g_\nu = g_\nu^T$ gives PMNS mixing matrix.

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$$(Y_E)^{ij}\,(\bar{L}^i\,H\,E_R^j) + (Y_\nu)^{ij}\,(\bar{L}^i\,\widetilde{H}\,\nu_R^j) + \frac{1}{2}\,M^{ij}\,(\nu_R^T)^i\,(\nu_R)^j + \text{h.c.}$$

  - Majorana mass term M breaks L-conservation.
  - Integrating out ν_R, yields dim-5 operator with

$$\frac{g_\nu}{\Lambda_{\not L}} = Y_\nu\,(M)^{-1}\,Y_\nu^T \qquad \textbf{(type-I see-saw)}$$

- Alternatively:
  - Additional heavy scalar triplets **(type-II see-saw)**
  - Additional heavy fermion triplets **(type-III see-saw)**
  - Additional *light* sterile neutrinos

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PontecorvoMakiNakagawaSakata Matrix, and Parameter Counting for dim-5 Term

- General parameterization: (no sterile neutrinos)

$$U_{PMNS} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

  - 3 mixing angles: θ_{12}, θ_{13}, θ_{23}
  - 1 "Dirac-phase": δ
  - 2 "Majorana-phases": α_1, α_2

- Corresponds to parameter counting from dim-5 term:

Quantity	Symbol	Moduli	Phases
Charged Yukawa Matrix	Y_E	9	9
Dim-5 Neutrino Matrix	$g_\nu = g_\nu^T$	6	6
Flavour Symmetry Group	$U(3)_L \times U(3)_{E_R}$	-6	-12
Physical Parameters:	Masses m_ℓ^i, m_ν^i	3 + 3	
	Angles θ_{ij}	3	
	CP phases:		3

Parameter Counting for See-Saw Models, [Broncano/Gavela/Jenkins hep-ph/0210271]

Quantity	Symbol	Moduli	Phases
Charged Yukawa Matrix	Y_E	9	9
Dirac Term for ν_R ($n' > 0$)	Y_ν	3 n'	3 n'
Majorana Mass Term	$M_\nu = M_\nu^T$	$\frac{n'(n'+1)}{2}$	$\frac{n'(n'+1)}{2}$
Flavour Symmetry Group	$U(3)_L \times U(3)_{E_R}$	-6	-12
	$U(n')_{\nu_R}$	$-\frac{n'(n'-1)}{2}$	$-\frac{n'(n'+1)}{2}$
Physical Parameters:	Masses m_ℓ^i, m_ν^i, M_ν^i	$3 + n' + n'$	
	Angles θ_{ij}	$2n'$	
	CP phases:		$3(n' - 1)$

- To access all high-energy flavour parameters of the see-saw model (relevant for leptogenesis), the dim-5 term in the low-energy theory is usually not sufficient. (Exception: Hierarchical Majorana masses.)

Example: $n' = 3$ Majorana Neutrinos

- Besides the three Majorana masses, the high-energy theory contains 3 additional angles and 3 additional phases.
- Parametrize in terms of orthogonal complex matrix $RR^T = 1$, such that

$$Y_\nu = \mathrm{diag}[\sqrt{M_\nu}]\, R\, \mathrm{diag}[\sqrt{m_\nu}]\, U^\dagger_{\mathrm{PMNS}}\, \langle H^0 \rangle\,,$$

$$\Rightarrow \quad g_\nu \propto Y_\nu^T\, \mathrm{diag}[M_\nu^{-1}]\, Y_\nu = U^*_{\mathrm{PMNS}}\, \mathrm{diag}[m_\nu]\, U^\dagger_{\mathrm{PMNS}}$$

(in a basis where Y_E and M_ν are diagonal). [Casas/Ibarra, hep-ph/0103065]

- Flavour couplings of dim-6 terms to be considered, too.
  - Highly suppressed at low-energies, IF $\Lambda = \Lambda_{\not L} \sim M_{\mathrm{GUT}}$.
  - L-conserving NP effects with $\Lambda = \Lambda_{\mathrm{LFV}} \gtrsim 1$ TeV may enter, too.

1.3. Origin of Flavour Hierarchies?

(a) Flavour Symmetries

- Idea:
  - Fermion families transform (differently) under some symmetry group.
  - Some spurion fields break symmetry.
- Examples:
  - Froggatt-Nielsen approach ($U(1)$ symmetry + heavy fermion messengers)
  - Discrete (non-Abelian) flavour symmetry groups (S^3, A_4, D_4, ...)

Flavour symmetries may also arise "accidentally", if flavour mixing dominated by exchange of some heavy messengers.

(b) Models with Extra Dimensions

- SM flavour hierarchies explained by displacement of fermion fields along the ED.
  (alternatively via power-law running of masses)
- Works both for models with
  - large (flat) extra dimensions (ADD)
  - small (warped) extra dimension (RS)
- (Exponential) suppression factors from wave-function overlaps in ED.
- FCNC by tree-level exchanges of KK excitations of neutral gauge bosons.
  (non-universal couplings due to different localizations)
- Violation of GIM cancellations from KK fermions in loops.

Theoretical Lessons:

- Different pattern of neutrino masses and mixings, compared to quarks.
- See-saw scenarios favoured.
- Related to breaking of (accidental) L-symmetry.
- Points towards scales $\gg M_W$, presumably GUT scale.
- (Discrete) flavour symmetries and/or ED may explain the observed hierarchies.
- ...

Open questions:

- Common origin of quark and lepton flavour ? — (SUSY-GUTs, ED, ... ?)
- Other sources of LFV from NP at the TeV scale ?
- Relevant leptogenesis parameters from low-energy observables ?
- ...

- Look for testable predictions/correlations for observables in different NP models. (SUSY, ED, Little Higgs, ...)
- Also: Model-independent approaches (MLFV).

2. LFV

- Study LFV processes with charged leptons (tiny in SM with massive ν's).

$$\text{e.g.} \qquad \bar{\ell}_i \sigma^{\mu\nu} \ell_j \, F^{\rm em}_{\mu\nu} \,, \qquad \bar{\ell}_i \Gamma^a \ell_j \, \bar{q}_k \Gamma_a q_\ell \,, \quad \bar{\ell}_i \Gamma^a \ell_j \, \bar{\ell}_k \Gamma_a \ell_\ell$$

- Also: LF-conserving CP-violating processes (tiny in SM with massive ν's).

$$\text{e.g.} \qquad \bar{\ell}_i \sigma^{\mu\nu} \gamma_5 \ell_i \, F^{\rm em}_{\mu\nu}$$

- Determine/constrain size of coefficients (relative to EW processes):

$$\epsilon \sim \frac{m_W^2}{\Lambda_{\rm NP}^2} \, \frac{g_{\rm NP}^2}{g_W^2} \, \delta_{\rm CP} \, \delta_{\rm mix}$$

Observable	Operator	Limit on ϵ
eEDM	$\overline{e_L} \sigma^{\mu\nu} \gamma_5 e_R F_{\mu\nu}$	$\lesssim 1.1 \times 10^{-3}$
$B(\mu \to e\gamma)$	$\overline{\mu} \sigma^{\mu\nu} e F_{\mu\nu}$	$\lesssim 1.4 \times 10^{-4}$
$B(\tau \to \mu\gamma)$	$\overline{\tau} \sigma^{\mu\nu} \mu F_{\mu\nu}$	$\lesssim 2.2 \times 10^{-2}$
$B(K_L^0 \to \mu^\pm e^\mp)$	$(\overline{\mu}\gamma^\mu P_L e)(\overline{s}\gamma^\mu P_L d)$	$\lesssim 2.9 \times 10^{-7}$

[M. Raidal et al., arXiv:0801.1826]

2.1. Phenomenological Status of LFV

Present bounds on (some) lepton-flavour violating decays: [BRs from PDG]*

$\mu^- \to e^- \nu_e \bar{\nu}_\mu$	:	$< 1.2\%$
$\mu^- \to e^- \gamma$	:	$< 1.2 \cdot 10^{-11}$
$\mu^- \to e^- e^+ e^-$	:	$< 1.0 \cdot 10^{-12}$
$\mu^- \to e^- \gamma\gamma$	:	$< 7.2 \cdot 10^{-11}$

$\pi^+ \to \mu^+ \bar{\nu}_e$	:	$< 8.0 \cdot 10^{-3}$
$\pi^0 \to \mu^\pm e^\mp$	:	$< 3.6 \cdot 10^{-10}$
$K^+ \to \pi^+ \mu^+ e^-$	:	$< 1.3 \cdot 10^{-11}$
$K_L^0 \to \mu^\pm e^\mp$	:	$< 4.7 \cdot 10^{x-12}$
$D^+ \to \pi^+ \mu^\pm e^\mp$	:	$< 3.4 \cdot 10^{-5}$
$D^0 \to \mu^\pm e^\mp$	:	$< 8.1 \cdot 10^{-7}$

$\tau^- \to e^- \gamma$	:	$< 1.1 \cdot 10^{-7}$
$\tau^- \to \mu^- \gamma$	:	$< 6.8 \cdot 10^{-8}$
$\tau^- \to e^- \pi^0$	:	$< 1.9 \cdot 10^{-7}$
$\tau^- \to \mu^- \pi^0$	:	$< 4.1 \cdot 10^{-7}$
$\tau^- \to e^- e^+ e^-$	:	$< 2.0 \cdot 10^{-7}$
$\tau^- \to \mu^- \mu^+ \mu^-$	:	$< 1.9 \cdot 10^{-7}$
...		

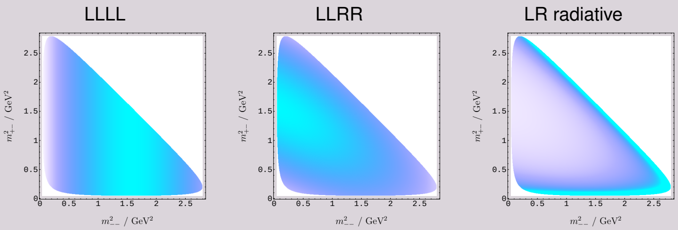
$B^+ \to \pi^+ e^\pm \mu^\mp$	:	$< 1.7 \cdot 10^{-7}$
$B^+ \to K^+ \mu^\pm \tau^\mp$	:	$< 7.7 \cdot 10^{-5}$
$B^+ \to K^{*+} e^\pm \mu^\mp$	:	$< 1.4 \cdot 10^{-7}$
$B^0 \to e^\pm \mu^\mp$	:	$< 9.2 \cdot 10^{-8}$
$B^0 \to e^\pm \tau^\mp$	:	$< 2.8 \cdot 10^{-5}$
$B^0 \to \mu^\pm \tau^\mp$	:	$< 2.2 \cdot 10^{-5}$
...		

* Bounds on 3-body decays depend on assumptions about phase-space distributions.
See e.g. discussion in [Dassinger/TF/Mannel/Turczyk, arXiv:0707.0988]

Classification of LFV observables

- Dipole transitions ($\mu \to e\gamma$, $\tau \to \mu(e)\gamma$)
  - also contributions from virtual photons to 3-lepton decays.
- Four-charged-lepton transitions ($\mu \to 3e$, $\tau \to 3\mu$, etc.)
  - Different chiralities involved.
  - Full-fledged analysis requires careful treatment of phase space.

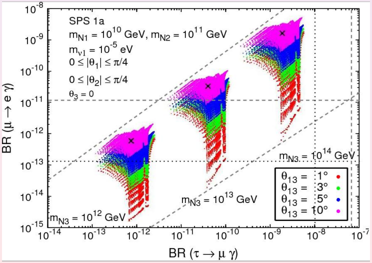
E.g. phase space in $\tau \to 3\mu$: [Dassinger et al. 07]



  - Also contribute at 1-loop to LFV Z^0-decays.
- 2-lepton–2-quark transition: (hadronic decays, ℓ-ℓ' conversions in nuclei)

(a) LFV in SUSY models

- Many LFV and CP-violating effects, extensively studied.
- Originate from misalignment between fermions and sfermions.
  Constrained to be small $\rightarrow$ mass insertion approximation.
- Predictions depend on assumptions on soft SUSY breaking and on the see-saw parameters.
- Example:
  Correlation between $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$,
  as a function of θ_{13} and the heaviest Majoran mass M_{N_3} for SPS 1a.

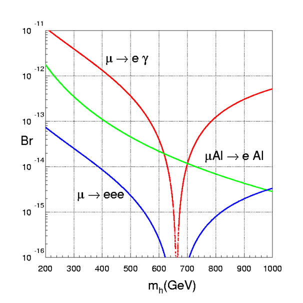


[Antusch et al, hep-ph/0607263]

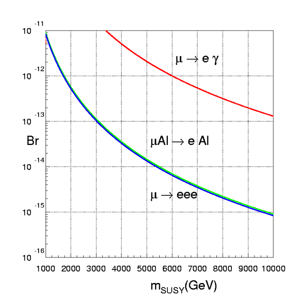
Higgs- vs. Gaugino-mediated LFV in SUSY

- Higgs-mediated LFV arises from loop-induced non-holomorphic couplings.
- Decoupling limit for Higgs-mediated or Gaugino-mediated LFV shows different correlations between different observables:

Higgs-mediated



Gaugino-mediated



$\tan_\beta = 50$ and $\delta_{LL}^{21} = 10^{-2}$

[Paradisi, hep-ph/0601100]

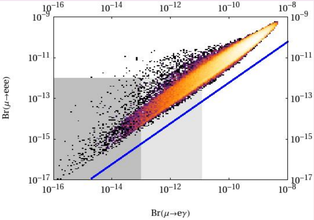
(b) Littlest Higgs Model with T-parity (LHT)

[Low; Hubisz et al.; Blanke et al.; Deandrea et al; . . . 2004+]

- New gauge bosons (detectable at the LHC).
- New doublets of mirror leptons (and quarks) with masses of order TeV.
  $\to$ Potential LFV effects exceeding the SM by many orders of magnitude.
- Phenomenology depends on:

LHT Scale parameter:	f ,	Mirror lepton masses:	M^ℓ_{H1} , M^ℓ_{H1} , M^ℓ_{H1} ,
Mirror lepton mixings:	θ^ℓ_{12} , θ^ℓ_{23} , θ^ℓ_{13} ,	New (Dirac) phases:	δ^ℓ_{12} , δ^ℓ_{23} , δ^ℓ_{13} ,

- Example: Correlation between $\mathcal{B}[\mu \to 3e]$ and $\mathcal{B}[\mu \to e\gamma]$:



using $f = 1$ TeV, and $300\ \mathrm{GeV} \leq M^\ell_{H_i} \leq 1.5\ \mathrm{TeV}$
blue dots: dipole contribution $\mu \to e\gamma^*$
[Blanke et al., arXiv:0906.5454]

$\to$ distinguishes LHT from MSSM (where $\mu \to e\gamma^*$ often dominates)
$\to$ mirror leptons must be quasi-degenerate or mixings very hierarchical (to fulfill bounds).

Comparison LHT and MSSM

Consider 3-lepton decays or $\mu - e$ conversion:

- In LHT dominated by Z^0-penguin and box diagrams.
- In MSSM typically dominated by dipole operator (or Higgs bosons).

ratio	LHT	MSSM (dipole)	MSSM (Higgs)
$\frac{Br(\mu^- \to e^- e^+ e^-)}{Br(\mu \to e\gamma)}$	$0.02 \ldots 1$	$\sim 6 \cdot 10^{-3}$	$\sim 6 \cdot 10^{-3}$
$\frac{Br(\tau^- \to e^- e^+ e^-)}{Br(\tau \to e\gamma)}$	$0.04 \ldots 0.4$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \to \mu^- \mu^+ \mu^-)}{Br(\tau \to \mu\gamma)}$	$0.04 \ldots 0.4$	$\sim 2 \cdot 10^{-3}$	$0.06 \ldots 0.1$
$\frac{Br(\tau^- \to e^- \mu^+ \mu^-)}{Br(\tau \to e\gamma)}$	$0.04 \ldots 0.3$	$\sim 2 \cdot 10^{-3}$	$0.02 \ldots 0.04$
$\frac{Br(\tau^- \to \mu^- e^+ e^-)}{Br(\tau \to \mu\gamma)}$	$0.04 \ldots 0.3$	$\sim 1 \cdot 10^{-2}$	$\sim 1 \cdot 10^{-2}$
$\frac{Br(\tau^- \to e^- e^+ e^-)}{Br(\tau^- \to e^- \mu^+ \mu^-)}$	$0.8 \ldots 2.0$	~ 5	$0.3 \ldots 0.5$
$\frac{Br(\tau^- \to \mu^- \mu^+ \mu^-)}{Br(\tau^- \to \mu^- e^+ e^-)}$	$0.7 \ldots 1.6$	~ 0.2	$5 \ldots 10$
$\frac{R(\mu\mathrm{Ti} \to e\mathrm{Ti})}{Br(\mu \to e\gamma)}$	$10^{-3} \ldots 10^{2}$	$\sim 5 \cdot 10^{-3}$	$0.08 \ldots 0.15$

[Blanke et al. 09]

• see also talk on “LFV in minimal see-saw models” by J. Kamenik (monday)

2.3. MFV in the Lepton Sector [Cirigliano et al, hep-ph/0507001, hep-ph/0601111]

- Non-observation of LFV decays puts severe constraints on NP parameters:
  - e.g. allowing for *generic* coupling constants for effective operators

$$\frac{1}{\Lambda_{\rm LFV}^2}\,\bar{L}^i \sigma^{\mu\nu} H E_R^j\, F_{\mu\nu}\,,$$

  the bound on $\mu \to e\gamma$ would imply $\Lambda_{LFV} > 10^5$ TeV.

  - In other words, allowing for NP at the TeV scale (from considerations of EWSB), requires specifically tuned LFV coefficients.

- MFV principle relates NP flavour coefficients to SM flavour structures.

  [see talk by Chr. Smith (today)]

- Complication compared to quark sector:
  - Neutrino masses themselves already constitute an extension of the SM.
  - Specification of neutrino-field content on top of MFV hypothesis.
  - Scale for lepton-*number* violation Λ_L to be distinguished from Λ_{LFV}.

$\Rightarrow$ Residual model-dependence for MLFV.

[see also: Davidson/Palorini, hep-ph/0607329]

MLFV as an Effective Theory (minimal field content)

- Lepton-flavour symmetry in minimally extended SM: $SU(3)_L \times SU(3)_{E_R}$
- Broken by flavour matrices,

$$\text{Dim-5:} \quad g_\nu \sim (\bar{6}, 1)\,, \qquad \text{Yukawa:} \quad Y_E \sim (3, \bar{3})\,.$$

- In the mass eigenbasis for charged leptons:

$$g_\nu = \frac{\Lambda_L}{v^2}\, U^*\, \mathrm{diag}[m_{\nu_i}]\, U^\dagger$$

- For the construction of 2- and 4-lepton operators, one needs

$$\Delta^i_j = (g_\nu^\dagger g_\nu)^i_j - \frac{1}{3}\delta^i_j\, \mathrm{tr}[g_\nu^\dagger g_\nu] \qquad \text{and} \qquad G^{kl}_{ij} = (g_\nu)_{ij}(g^*_\nu)^{kl} - \text{[trace terms]}$$

  appearing as the octet and 27-plet in $\bar{6} \times 6 = 1 + 8 + 27\,.$
- The leading effect due to $\Delta m^2_{\mathrm{atm}}$ can be singled out by using a non-linear representation of MLFV. [TF/Mannel, arXiv:0806.0717]

Non-linear Representation of Lepton-Flavour Symmetry [TF/Mannel, arXiv:0806:0717]

normal hierarchy: $SU(3)_L \times U(1)_L \to U(2)_L \times Z_2$

$$\langle g_\nu \rangle \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & g \end{pmatrix}, \qquad g \simeq \frac{\Lambda_{\not L}}{v^2} \sqrt{\Delta m^2_{\rm atm}}$$

inverted hierarchy: $SU(3)_L \times U(1)_L \to SO(2)_L \times U(1)_{L_3}$

$$\langle g_\nu \rangle \simeq \begin{pmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad g \simeq \frac{\Lambda_{\not L}}{v^2} \sqrt{\Delta m^2_{\rm atm}}$$

no hierarchy: $SU(3)_L \times U(1)_L \to SO(3)_L \to SO(2)_L \times Z_2$

$$\langle g_\nu \rangle \simeq \begin{pmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g \end{pmatrix} + \frac{\Lambda}{\Lambda_{\not L}} \begin{pmatrix} -\check{g} & 0 & 0 \\ 0 & -\check{g} & 0 \\ 0 & 0 & 2\check{g} \end{pmatrix}, \quad g \simeq \frac{\Lambda_{\not L}}{v^2} \overline{m}_\nu \,, \quad \frac{\Delta m^2_{\rm atm}}{\overline{m}^2_\nu} \sim \frac{\Lambda}{\Lambda_{\not L}}$$

- broken generators → Goldstone modes (considered unphysical).
- $\Delta m^2_{\rm sol}$ and U_{PMNS} from spurions of residual flavour symmetry.

LFV Operators in MLFV

- Dim-6 LFV operators in the minimally extended SM + MFV (linear in y_i)

$$
\begin{aligned}
O_{LL}^{(1)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu L^j)\; H^\dagger i D_\mu H\,, \\
O_{LL}^{(2)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu \tau^a L^j)\; H^\dagger \tau^a i D_\mu H\,, \\
O_{LL}^{(3)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu L^j)\; (\bar{Q}_L \gamma_\mu Q_L)\,, \\
O_{LL}^{(4d)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu L^j)\; (\bar{D}_R \gamma_\mu D_R)\,, \\
O_{LL}^{(4u)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu L^j)\; (\bar{U}_R \gamma_\mu U_R)\,, \\
O_{LL}^{(5)} &= \Delta^i_j \; (\bar{L}_i \gamma^\mu \tau^a L^j)\; (\bar{Q}_L \gamma_\mu \tau^a Q_L)
\end{aligned}
$$

$$
\begin{aligned}
O_{RL}^{(1)} &= (Y_E^\dagger \Delta)^i_j \; g' H^\dagger\, (\bar{E}_{Ri} \sigma^{\mu\nu} L^j)\, B_{\mu\nu}\,, \\
O_{RL}^{(2)} &= (Y_E^\dagger \Delta)^i_j \; g H^\dagger\, (\bar{E}_{Ri} \sigma^{\mu\nu} \tau^a L^j)\, W^a_{\mu\nu}\,, \\
O_{RL}^{(3)} &= (Y_E^\dagger \Delta)^i_j \; (D_\mu H)^\dagger\, (\bar{E}_{Ri} D_\mu L^j)\,, \\
O_{RL}^{(4)} &= (Y_E^\dagger \Delta)^i_j \; (\bar{E}_{Ri} L^j)\, (\bar{Q}_L Y_D D_R)\,, \\
O_{RL}^{(5)} &= (Y_E^\dagger \Delta)^i_j \; (\bar{E}_{Ri} \sigma^{\mu\nu} L^j)\, (\bar{Q}_L \sigma_{\mu\nu} Y_D D_R)\,, \\
O_{RL}^{(6)} &= (Y_E^\dagger \Delta)^i_j \; (\bar{E}_{Ri} L^j)\, (\bar{U}_R Y_U^\dagger i\tau^2 Q_L)\,, \\
O_{RL}^{(7)} &= (Y_E^\dagger \Delta)^i_j \; (\bar{E}_{Ri} \sigma^{\mu\nu} L^j)\, (\bar{U}_R \sigma_{\mu\nu} Y_U^\dagger i\tau^2 Q_L)\,.
\end{aligned}
$$

for 2-lepton processes, and

$$
O_{LLRR}^{(4)} = \Delta^i_j \; (\bar{L}_i \gamma^\mu L^j)\; (\bar{E}_R \gamma_\mu E_R)\,,
$$

$$
\begin{aligned}
O_{LLLL}^{(3)} &= \left(\Delta^i_j \delta^{kl} + c\, G^{ik}_{jl}\right) (\bar{L}_i \gamma^\mu L^j)\; (\bar{L}_k \gamma_\mu L^l)\,, \\
O_{LLLL}^{(5)} &= \left(\Delta^i_j \delta^{kl} + c'\, G^{ik}_{jl}\right) (\bar{L}_i \gamma^\mu \tau^a L^j)\; (\bar{L}_k \gamma_\mu \tau^a L^l)
\end{aligned}
$$

for 4-lepton processes.

[Cirigliano et al, hep-ph/0507001]
[Dassinger/TF/Mannel/Turczyk, arXiv:0707.0988]

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for 2-lepton processes, and

$$O_{LLRR}^{(4)} = \Delta^i_j \, (\bar L_i \gamma^\mu L^j) \; (\bar E_R \gamma_\mu E_R) \,,$$

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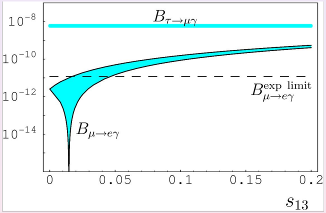
for 4-lepton processes.

[Cirigliano et al, hep-ph/0507001]
[Dassinger/TF/Mannel/Turczyk, arXiv:0707.0988]

MLFV Phenomenology (minimal field content)

- LFV decay rates are sizeable, only if $\Lambda_{\not L} \gg \Lambda_{\rm LFV}$,
  Example: $\mathcal{B}(\mu \to e\gamma) > 10^{-13}$ requires $\Lambda_{\not L} > 10^9 \cdot \Lambda_{\rm LFV}$.
- $\Lambda_{\not L}$ drops out in *ratios* of LFV decays, e.g. $\mathcal{B}(\mu \to e\gamma)/\mathcal{B}(\tau \to \mu\gamma) \sim 10^{-2} - 10^{-3}$



using $\Lambda_{\not L} = 10^{10} \cdot \Lambda_{\rm LFV}$
[Cirigliano/Grinstein/Isidori/Wise, hep-ph/0507001]

$\Rightarrow$ Better experimental prospects to observe $\mu \to e\gamma$ than $\tau \to \mu\gamma$.

- LFV decays of light hadrons very small within MLFV.
- Extending the field content, the above conclusions are relaxed, but the MLFV framework becomes less predictive.

Summary

- Hierarchies of lepton masses and mixings represent new challenges for the understanding of flavour in the context of more fundamental theories (GUTs, ED, . . .)

- LFV decays provide sensitive probe of New Physics models, complementary to the direct searches at the LHC. (SUSY, LHT, . . .)

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- Hierarchies of lepton masses and mixings represent new challenges for the understanding of flavour in the context of more fundamental theories (GUTs, ED, . . .)

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Striking New Physics Result observed at Kazimierz ?

conference dinner, restaurant "Kwadrans"

- New Physics interpretation: kwadrans =
- Old Physics interpretation: kwadrans =

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- New Physics interpretation: kwadrans $=$ quadron $= \frac{1}{2}$ (quark $+$ hadron)
- Old Physics interpretation: kwadrans $=$

Summary

- Hierarchies of lepton masses and mixings represent new challenges for the understanding of flavour in the context of more fundamental theories (GUTs, ED, . . .)

- LFV decays provide sensitive probe of New Physics models, complementary to the direct searches at the LHC. (SUSY, LHT, . . .)

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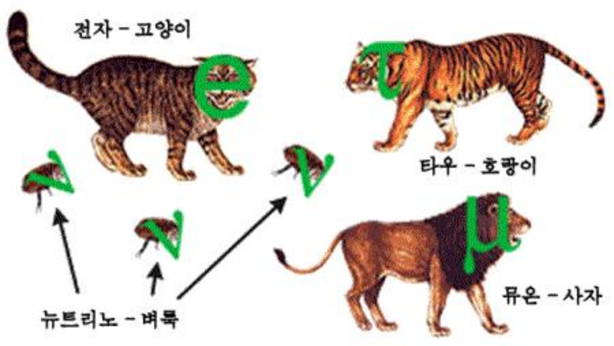
- New Physics interpretation: kwadrans =
- Old Physics interpretation: kwadrans = quadrant

. . . still many puzzles remain . . .



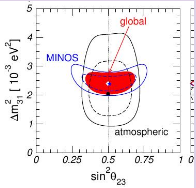
Backup Slides

Experimental evidences

- Neutrino oscillations: Flavour eigenstates $\neq$ Mass eigenstates
  - atmospheric neutrinos (+ accelerator exp.):

$$|\Delta m^2_{\rm atm}| \simeq |\Delta m^2_{23}| \simeq |\Delta m^2_{13}| = (2.40^{+0.12}_{-0.11}) \times 10^{-3}{\rm eV}^2\,,$$

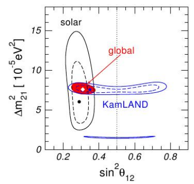
$$\sin^2\theta_{\rm atm} \simeq \sin^2\theta_{23} = 0.50^{+0.07}_{-0.06}\,.$$



  - solar neutrinos (+ reactor experiments):

$$\Delta m^2_{\odot} \simeq |\Delta m^2_{12}| = (7.65^{+0.23}_{-0.20}) \times 10^{-5}{\rm eV}^2\,,$$

$$\sin^2\theta_{\odot} \simeq \sin^2\theta_{12} = 0.304^{+0.022}_{-0.016}\,.$$



[Schwetz/Tortola/Valle, arXiv:0808.2016]

Discrete Flavour Groups: Example A_4 [see e.g., Ma/Rajasekaran, hep-ph/0106291]

- Symmetry group of even permutations (tetrahedron), with $\frac{1}{2}\,4! = 12$ generators.
- 4 irreducible representation: $\underline{1}, \underline{1}', \underline{1}'', \underline{3}$. Singlets from $\underline{3} = (a, b, c)$ via

$$\underline{1} = a_1 a_2 + b_1 b_2 + c_1 c_2\,,$$
$$\underline{1}' = a_1 a_2 + \omega^2\, b_1 b_2 + \omega\, c_1 c_2\,,$$
$$\underline{1}'' = a_1 a_2 + \omega\, b_1 b_2 + \omega^2\, c_1 c_2\,, \qquad \text{with } \omega = e^{2i\pi/3}$$

- Simplest assignment of quantum numbers

$$\begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix}_i \sim \underline{3}\,, \qquad (\ell_L^c)_1 \sim \underline{1}\,, \quad (\ell_L^c)_2 \sim \underline{1}'\,, \quad (\ell_L^c)_3 \sim \underline{1}''\,,$$

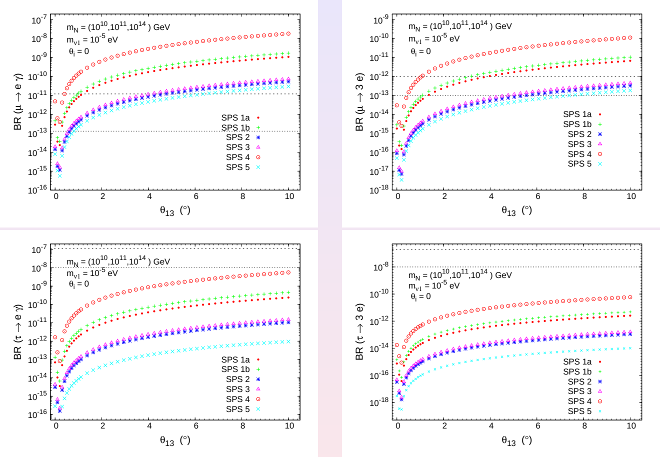
together with three Higgs doublets $\Phi_i \sim \underline{3}$ with $\langle \phi^0 \rangle_i = v_i \equiv v\,.$

- Charged-lepton Yukawa term $\left(h_{ijk}\, \bar{L}_i\, \ell_j^c\, \Phi_k + \text{h.c.}\right)$ yields mass matrix

$$v \begin{pmatrix} h_1 & h_2 & h_3 \\ h_1 & h_2\omega^2 & h_3\omega \\ h_1 & h_2\omega & h_3\omega^2 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} \sqrt{3} v h_1 & 0 & 0 \\ 0 & \sqrt{3} v h_2 & 0 \\ 0 & 0 & \sqrt{3} v h_3 \end{pmatrix}$$

- Together with (softly breaking) mass term for Majorana neutrinos, observed patterns in neutrino masses and mixings can be reproduced.

$\mu(\tau) \to e\gamma$ and $\mu(\tau) \to 3e$ for different SPS points



$B(\mu \to e\gamma)$ and $B(\mu \to 3e)$ as a function of θ_{13} (in degrees), for SPS 1a (dots), 1b (crosses), 2 (asterisks), 3 (triangles), 4 (circles) and 5 (times). A dashed (dotted) horizontal line denotes the present experimental bound (future sensitivity).

[Raidal et al. 08]

LFV bounds in LHT model

[Blanke et al. 09]

decay	$f = 1000$	$f = 500$	SuperB sensitivity
$\tau \to e\gamma$	$8 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$2 \cdot 10^{-9}$
$\tau \to \mu\gamma$	$8 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$2 \cdot 10^{-9}$
$\tau^- \to e^- e^+ e^-$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$2 \cdot 10^{-10}$
$\tau^- \to \mu^- \mu^+ \mu^-$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$2 \cdot 10^{-10}$
$\tau^- \to e^- \mu^+ \mu^-$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	
$\tau^- \to \mu^- e^+ e^-$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	
$\tau^- \to \mu^- e^+ \mu^-$	$6 \cdot 10^{-14}$	$1 \cdot 10^{-13}$	
$\tau^- \to e^- \mu^+ e^-$	$6 \cdot 10^{-14}$	$1 \cdot 10^{-13}$	
$\tau \to \mu\pi$	$4 \cdot 10^{-10}$	$5 \cdot 10^{-8}$	
$\tau \to e\pi$	$4 \cdot 10^{-10}$	$5 \cdot 10^{-8}$	
$\tau \to \mu\eta$	$2 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$4 \cdot 10^{-10}$
$\tau \to e\eta$	$2 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	$6 \cdot 10^{-10}$
$\tau \to \mu\eta'$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	
$\tau \to e\eta'$	$1 \cdot 10^{-10}$	$2 \cdot 10^{-8}$	