Kazimierz, July 2009

Minimal Flavor Violation in supersymmetric theories

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- Outline

Introduction: SM and MSSM flavor structures

I. Minimal Flavor Violation (MFV)

II. CP-violation under MFV

III. RGE behavior of MFV

IV. MFV instead of R-parity?

V. How to test MFV?

Conclusion

Introduction

A. The flavor structures of the Standard Model

The three generations of quarks/leptons have *identical gauge interactions*

→ *flavor symmetry*: $G_f = U(3)^5 = U(3)_Q \times U(3)_U \times U(3)_D \times U(3)_L \times U(3)_E$

Chivukula, Georgi '87

- The only sources of breaking are the *Yukawa couplings*:

$$\mathcal{L}_{Yukawa} = U\mathbf{Y}_u QH + D\mathbf{Y}_d QH^\dagger + E\mathbf{Y}_e LH^\dagger$$

Very special: hierarchical masses and CKM matrix, unique CP-phase.

- *No flavor mixing in the lepton sector* (no ν_R and ν_L massless).

- *Baryon ($\mathcal{B}$) and lepton numbers ($\mathcal{L}$)* are conserved in the SM Lagrangian.

B. Warm-up: "MFV" in the Standard Model

- The SM is made *invariant under* G_f by forcing the Yukawas to transform as:

$$\mathbf{Y}_u \to g_U \mathbf{Y}_u g_Q^\dagger, \quad \mathbf{Y}_d \to g_D \mathbf{Y}_d g_Q^\dagger, \quad \mathbf{Y}_e \to g_E \mathbf{Y}_e g_L^\dagger$$ (= *spurions*)

Since then $\mathcal{L}_{Yukawa} = U\mathbf{Y}_u QH + D\mathbf{Y}_d QH^\dagger + E\mathbf{Y}_e LH^\dagger \overset{U(3)^5}{\longrightarrow} \mathcal{L}_{Yukawa}$

Background values: $v\mathbf{Y}_u = m_u V_{CKM}$, $v\mathbf{Y}_d = m_d$, $v\mathbf{Y}_e = m_e$.

- All *SM amplitudes must then be invariant under* G_f , at all orders.

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Example: $d_L^I \to d_L^J \gamma^*$ from $\mathcal{O}_\gamma \sim \bar{Q}\gamma_\nu Q\, D_\mu F^{\mu\nu}$

Simplest flavor-violating operator: $\mathcal{O}_\gamma \sim \bar{Q}\gamma_\nu \mathbf{Y}_u^\dagger \mathbf{Y}_u Q\, D_\mu F^{\mu\nu}$

Reproduces leading GIM-breaking $\mathbf{Y}_u^\dagger \mathbf{Y}_u \sim m_t^2 \begin{pmatrix} |V_{td}|^2 & V_{td}^* V_{ts} & V_{td}^* V_{tb} \\ V_{ts}^* V_{td} & |V_{ts}|^2 & V_{ts}^* V_{tb} \\ V_{tb}^* V_{td} & V_{tb}^* V_{ts} & |V_{tb}|^2 \end{pmatrix}$

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Suppressed by a power $\sim \dfrac{\mathrm{m}_{d^I} \mathrm{m}_{d^J}}{v^2}$ compared to $d_L^I \to d_L^J \gamma^*$.

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Example: $\ell^I \to \ell^J \gamma^*$

No matter the operator, $\bar{L}\gamma_\nu L D_\mu F^{\mu\nu}, E\gamma_\nu \bar{E} D_\mu F^{\mu\nu}, E\sigma_{\mu\nu} L HF^{\mu\nu},...$

Always flavor-conserving, since $\mathbf{Y}_e$ is diagonal (no LFV).

C. The flavor structures of the MSSM

Squarks and sleptons are *scalar flavored particles*.

$\Rightarrow$ This leads to many problems, the so-called *MSSM flavor puzzles*.

- MSSM gauge interactions still exhibit the $U(3)^5$ *flavor symmetry*.

- *Many new flavor couplings* $\Leftrightarrow$ new spurions, a priori not hierarchical.

- *New contributions* to flavor transitions

e.g.: $\mathcal{L}_{MSSM} \supset \tilde{Q}^{\dagger}\mathbf{m}_Q^2\tilde{Q} \quad \rightarrow \quad \mathcal{O}_{\gamma} \sim \bar{Q}\gamma_{\nu}\mathbf{m}_Q^2 Q\, D_{\mu}F^{\mu\nu}$

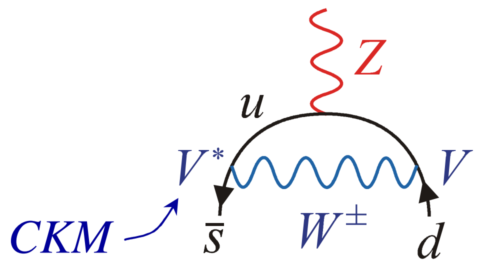
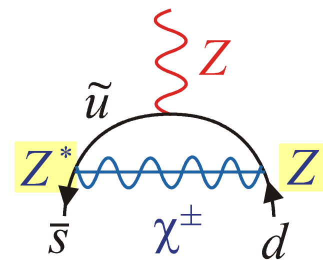
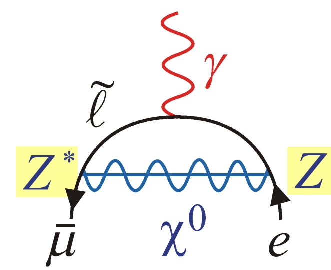
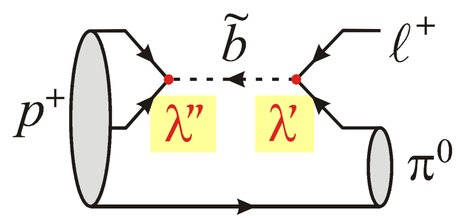
$\mathcal{L}_{MSSM} \supset \tilde{L}^{\dagger}\mathbf{m}_L^2\tilde{L} \quad \rightarrow \quad \mathcal{O}_{\mu e\gamma} \sim (E\mathbf{Y}_e\mathbf{m}_L^2\sigma_{\mu\nu}L)\, H_d F^{\mu\nu}$

- Experimental data impose to *fine-tune those additional spurions*:

Must be ~ aligned with those of the SM: $\mathbf{m}_Q^2 \sim \mathbf{Y}_u^{\dagger}\mathbf{Y}_u + \ldots,\ \mathbf{m}_L^2 \sim \mathbf{1} + \ldots$.

C. The flavor structures of the MSSM

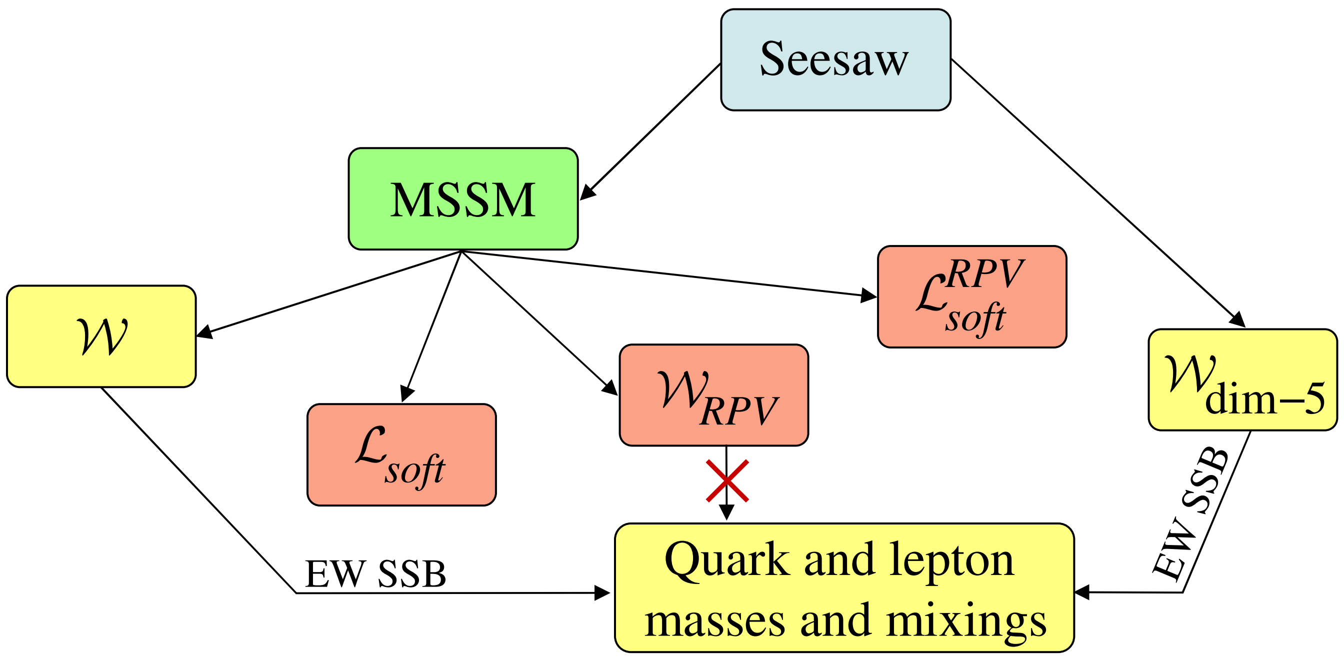
The three classes of flavored fine-tunings:

	SM		*MSSM*	*Origin*
Squark: FCNCs		≪		Soft-breaking terms (squark masses)
Slepton: LFVs	~ absent	≪		Soft-breaking terms (slepton masses)
$\Delta\mathcal{B}$ and $\Delta\mathcal{L}$ Proton decay	absent	≪		R-parity violating couplings

All these fine-tunings risk ruining the MSSM naturality.

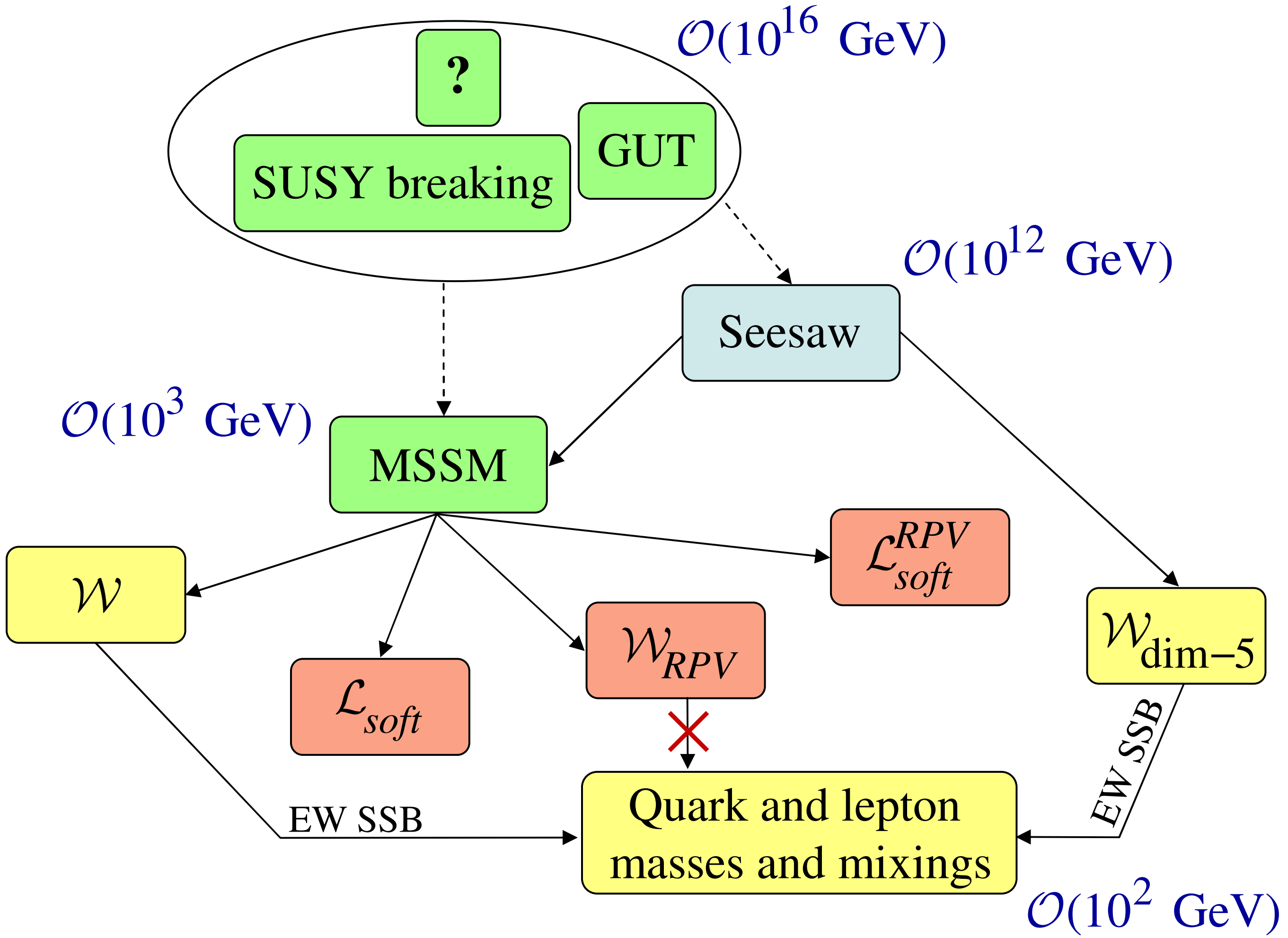
I. The MFV hypothesis

A. MFV and the origin of the flavor structures:



Only the flavor-breakings in the SM fermionic sector have been probed experimentally.

A. MFV and the origin of the flavor structures:



The MSSM is not the ultimate theory, but only a “low-energy” effective theory.

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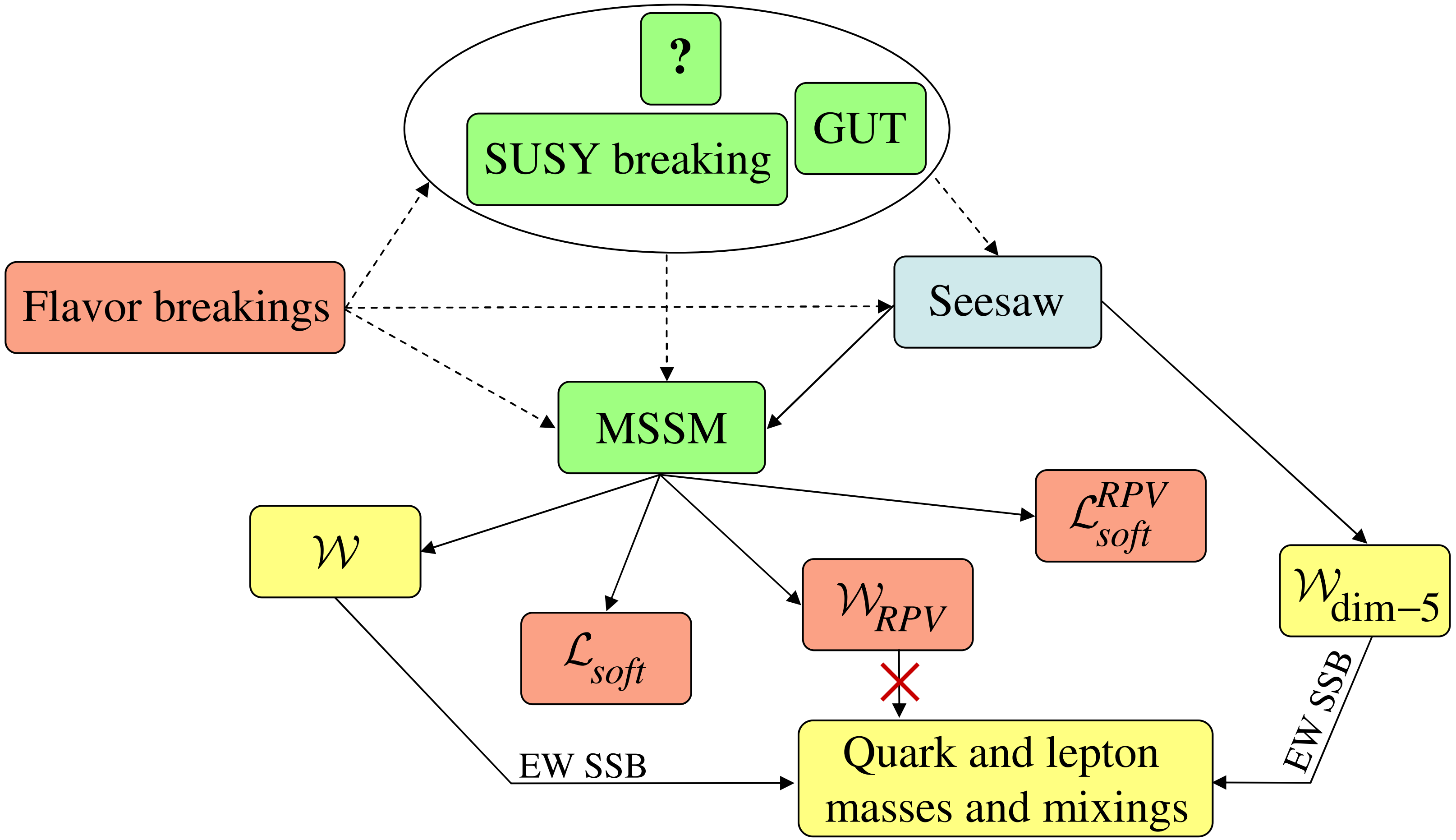

Some mechanism beyond the MSSM must explain the *origin of all flavor structures*.

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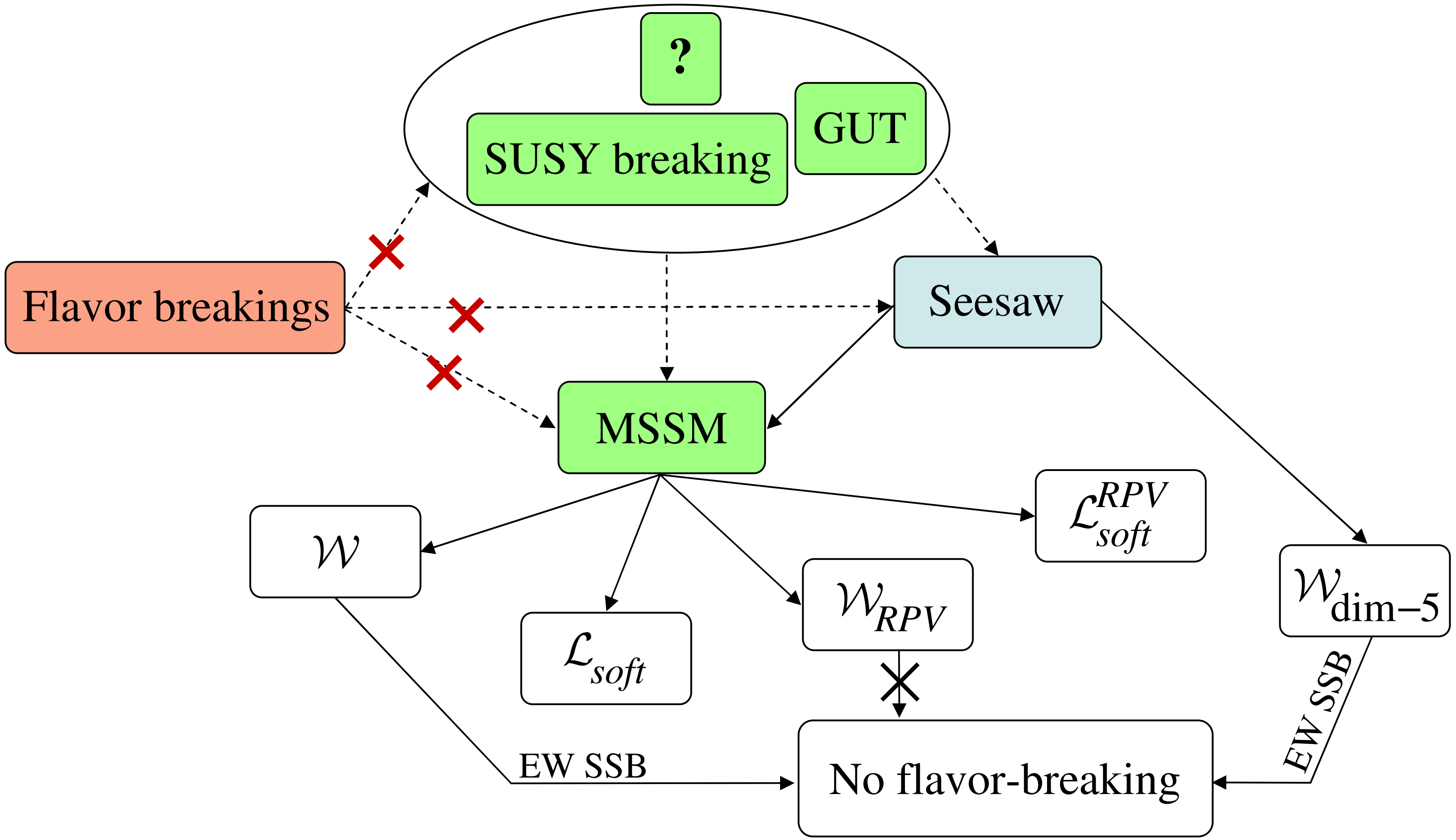

If this mechanism is turned off, flavor-breaking terms become forbidden.

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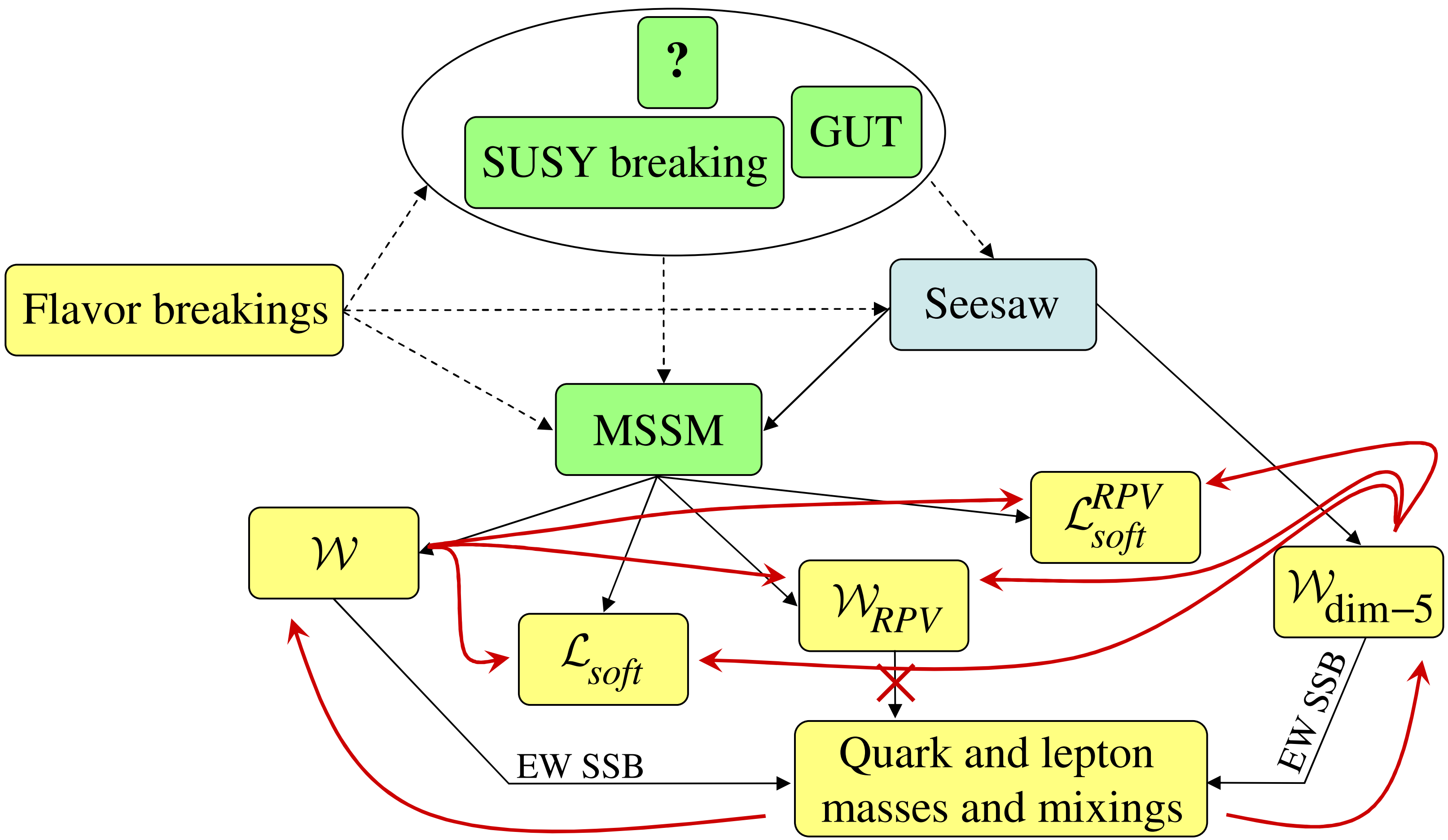


With MFV, all the flavor-breaking couplings are reconstructed in terms of the fermion masses and mixings, and become *naturally hierarchical*.

B. In practice:

- *Minimality hypothesis:* Minimal spurion content allowing for the known fermion masses and mixing - *this is the essence of MFV!*

Essentially, ~ to the Yukawas $\mathrm{Y}_u, \mathrm{Y}_d, \mathrm{Y}_e$ plus a few seesaw spurions.

- *Symmetry principle*: All Lagrangian couplings written as formal G_f -invariants

$$\mathbf{m}_Q^2 = m_0^2 (a_0 1 + a_1 \mathrm{Y}_u^\dagger \mathrm{Y}_u + a_2 \mathrm{Y}_d^\dagger \mathrm{Y}_d + \ldots) \quad \text{with} \quad a_i \sim \mathcal{O}(1)$$

←*naturality*

- *Freezing of the spurions* at their physical values:

Hall,Randall '90
D'Ambrosio,Giudice, Isidori,Strumia '02

$$\mathbf{m}_Q^2 \sim m_0^2 \left(\begin{pmatrix} 1 & 10^{-4} & 10^{-3} \\ 10^{-4} & 1 & 10^{-2} \\ 10^{-3} & 10^{-2} & 1 \end{pmatrix} + i \begin{pmatrix} 0 & 10^{-4} & 10^{-3} \\ 10^{-4} & 0 & 10^{-4} \\ 10^{-3} & 10^{-4} & 0 \end{pmatrix} \right)$$

These hierarchies come entirely from those of $\mathrm{Y}_u, \mathrm{Y}_d$.

C. MFV expansions in the quark sector

Hall,Randall '90, D'Ambrosio,Giudice, Isidori,Strumia '02, Colangelo,Nikolidakis,C.S. '08

- Only a *finite number* of terms thanks to Cayley-Hamilton identity:

$$X^3 - \langle X\rangle X^2 + \tfrac{1}{2} X(\langle X\rangle^2 - \langle X^2\rangle) = \tfrac{1}{3}\langle X^3\rangle - \tfrac{1}{2}\langle X\rangle\langle X^2\rangle + \tfrac{1}{6}\langle X\rangle^3$$

- Use the large *mass hierarchy* to set $(Y_i^\dagger Y_i)^2 \sim Y_i^\dagger Y_i$, leaving:

$$A \equiv Y_u^\dagger Y_u$$
$$B \equiv Y_d^\dagger Y_d$$

$$\mathbf{m}_Q^2 = m_0^2(a_1 1 + a_2 A + a_3 B + a_4\{A,B\} + b_1 i[A,B])$$

$$\mathbf{m}_U^2 = m_0^2(a_5 1 + Y_u(a_6 1 + a_7 B + a_8\{A,B\} + b_2 i[A,B])Y_u^\dagger)$$

$$\mathbf{m}_D^2 = m_0^2(a_9 1 + Y_d(a_{10} 1 + a_{11} A + a_{12}\{A,B\} + b_3 i[A,B])Y_d^\dagger)$$

$$\mathbf{A}_u = A_0 Y_u(c_1 1 + c_2 A + c_3 B + c_4\{A,B\} + d_1 i[A,B])$$

$$\mathbf{A}_d = A_0 Y_d(c_5 1 + c_6 A + c_7 B + c_8\{A,B\} + d_2 i[A,B])$$

Using these identities, all operators can be written as hermitian,

hence $a_i, b_i \in \mathbb{R},\ c_i, d_i \in \mathbb{C}$ since scalar mass terms are hermitian.

D. MFV expansions in the lepton sector

- Integrating out the right-handed (s)neutrinos:

Cirigliano,Grinstein Isidori,Wise '05

$$\mathrm{Y}_e\,, \qquad \mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu\,, \qquad \mathrm{Y}_\nu^T \mathrm{M}^{-1} \mathrm{Y}_\nu\,, \qquad \mathrm{Y}_\nu^\dagger \mathrm{M}^{-1*} \mathrm{M}^{-1} \mathrm{Y}_\nu\,, \quad \ldots$$

Lepton masses:

$$v_d \mathrm{Y}_e = \mathrm{m}_e$$

Neutrino masses:

$$v_u^2 \mathrm{Y}_\nu^T \mathrm{M}^{-1} \mathrm{Y}_\nu = U^* \mathrm{m}_\nu U^\dagger$$

Not completely fixed (we take $\mathrm{M} = M_R \mathbf{1}$):

$$v_u^2 \mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu = M_R U^* \mathrm{m}_\nu^{1/2} e^{2i\Phi} \mathrm{m}_\nu^{1/2} U^\dagger\,, \quad \Phi^{IJ} = \varepsilon^{IJK} \phi_K$$

Casas,Ibarra '01, Pascoli,Petcov, Yaguna '03,...

- More terms remain since there is no third-generation dominance for ν_L:

$$\mathbf{m}_L^2 = m_0^2 (a_1 \mathbf{1} + a_2 \mathrm{A} + a_3 \mathrm{B} + a_4 \mathrm{B}^2 + a_5 \{\mathrm{A},\mathrm{B}\} + a_6 \mathrm{BAB} + b_1 i[\mathrm{A},\mathrm{B}] + b_2 i[\mathrm{A},\mathrm{B}^2] + b_3 i(\mathrm{BAB}^2 - \mathrm{B}^2\mathrm{AB}))$$

$$\mathrm{A} \equiv \mathrm{Y}_e^\dagger \mathrm{Y}_e \qquad \mathrm{B} \equiv \mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu$$

Similar for $\mathbf{m}_E^2$ and $\mathbf{A}_e$.

Mercolli,C.S. '09

II. CP-violation under MFV

A. CP-violating phases in the MFV approach

Mercolli,C.S. '09

In the SM, CP-violation comes entirely from the phases in the spurions.

One in Y_u (Dirac), six in $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu$ (1 Dirac, 2 Majorana, 3 from the ϕ_K)

With MFV, are the only CP-violating phases those present in the spurions?

Conceptual reasons for *additional CP-phases*:

- The $U(3)^5$ does not say anything about CP-violating phases,

  All the *MFV coefficients are free complex parameters*.

- There can be new *CP-violating phases in other sectors*,

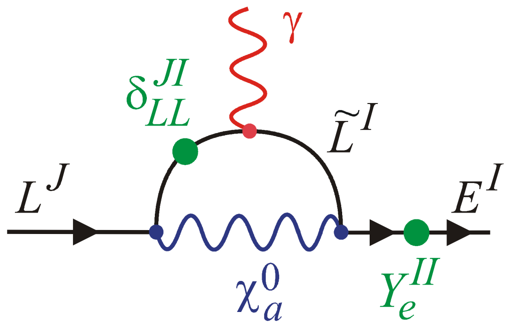
  CP-violation is a flavored phenomenon only in the SM!

- Potentially complex traces $\langle \mathrm{A}^l \mathrm{B}^m \mathrm{A}^n ... \rangle$ are $U(3)^5$ singlets,

  Absorbed in the coefficients: forcing them to stay real is a *fine-tuning*!

B. Consequence: Is MFV breaking down?

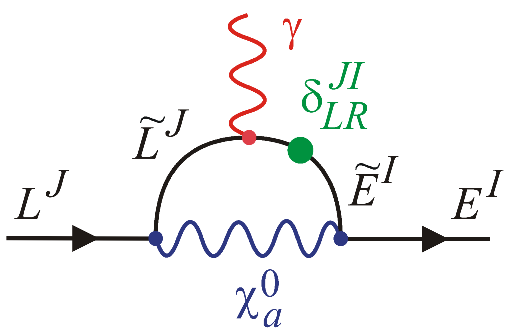
MFV is very effective to *constrain flavor transitions* like $\ell^I \to \ell^J$ or $d^I \to d^J$:



$$B(\ell^I \to \ell^J \gamma) \sim \frac{\alpha M_W^4 \tan^2\beta}{M_{SUSY}^8} \left| (\mathbf{m}_L^2)^{JI} + \ldots \right|^2$$

Correlations?

But does it limit *flavor-diagonal* operators → not too large EDMs?



Beyond MFV

$$\frac{d_I}{e} \sim \frac{\alpha}{M_{SUSY}^3} \operatorname{Im}\left(m_\ell^I \mu \tan\beta - v_d \mathbf{A}_e^{*II} \right) + \ldots$$

Diagonal part of the trilinear terms.

C. Classification of the CP-phases

MFV expansions, with $a_i, b_i \in \mathbb{R}$, $c_i, d_i \in \mathbb{C}$:

$$\mathbf{m}_L^2 = m_0^2(a_1 1 + a_2 \mathrm{A} + a_3 \mathrm{B} + a_5\{\mathrm{A},\mathrm{B}\} + a_6 \mathrm{BAB} + b_1 i[\mathrm{A},\mathrm{B}] + \mathcal{O}(\mathrm{A}^2,\mathrm{B}^2)),$$

$$\mathbf{m}_E^2 = m_0^2(a_7 1 + \mathrm{Y}_e(a_8 1 + a_9 \mathrm{B} + a_{11}\{\mathrm{A},\mathrm{B}\} + b_4 i[\mathrm{A},\mathrm{B}])\mathrm{Y}_e^\dagger + \mathcal{O}(\mathrm{A}^2,\mathrm{B}^2)),$$

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$\mathrm{A} \equiv \mathrm{Y}_e^\dagger \mathrm{Y}_e$

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In the squark sector: 13 CP-violating coefficients + 1 spurion phase

In the slepton sector: 15 CP-violating coefficients + 6 spurion phases

$\Rightarrow$ *Plenty of new CP-phases in MFV!*

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Flavor-blind phase: $\mathrm{Im}\, c_1$ (remember $d_I \sim \mathrm{Im}\, \mathbf{A}_e^{*II} \sim \mathrm{Im}\, c_1$)

Defined relative to the flavor-blind parameters of the MSSM ($\mu, M_1, M_2, ...$)

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Flavor-blind phase: $\mathrm{Im}\, c_1$

Defined relative to the flavor-blind parameters of the MSSM ($\mu, M_1, M_2, \ldots$)

Flavor-diagonal phases: $\mathrm{Im}\, c_{2-6}$ (remember $d_I \sim \mathrm{Im}\, \mathbf{A}_e^{*II} \sim \mathrm{Im}\, c_{2-6}$)

Contribute to EDMs at leading order in the MIA.

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Flavor-blind phase: $\mathrm{Im}\, c_1$

Defined relative to the flavor-blind parameters of the MSSM ($\mu, M_1, M_2, ...$)

Flavor-diagonal phases: $\mathrm{Im}\, c_{2-6}$

Contribute to EDMs at leading order in the MIA.

Flavor off-diagonal phases: $b_i, \mathrm{Re}\, d_i$, six phases of $\mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu$ ⟵ (hermitian op.)

Start to contribute to EDMs at 2nd order in the MIA ($d_I \sim \mathrm{Im}(\mathbf{m}_L^2)^{IK}(\mathbf{A}_e)^{KI}$).

D. Impact on the EDMs and LFV processes

Only a single operator dominates for $\mu \to e\gamma$ (coming from δ_{LL}):

$$B(\mu \to e\gamma) \sim \frac{\alpha M_W^4 \tan^2\beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} \left(Y_\nu^\dagger Y_\nu\right)^{12} \right|^2$$

Only a single operator per type of phases dominates for d_e:

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left(\frac{\mathrm{Im}\, c_1}{a_1 a_7} + \frac{\mathrm{Im}\, c_3}{a_1 a_7} Y_\nu^\dagger Y_\nu - \frac{b_1 \mathrm{Re}\, c_3}{a_1^2 a_7} [Y_\nu^\dagger Y_\nu, Y_e^\dagger Y_e] Y_\nu^\dagger Y_\nu + \ldots \right)^{11}$$

Flavor-blind — Flavor-diagonal — Flavor off-diagonal ($\geq$ neutrino phases)

Remark: $m_L^2 \approx m_0^2 a_1$, $m_R^2 \approx m_0^2 a_7$

D. Impact on the EDMs and LFV processes

Only a single operator dominates for $\mu \to e\gamma$ (coming from δ_{LL}):

$$B(\mu \to e\gamma) \sim \frac{\alpha M_W^4 \tan^2\beta}{M_{SUSY}^4} \left| \frac{a_3}{a_1} \frac{M_R \Delta m_{21}}{v_u^2} \right|^2$$

Only a single operator per type of phase dominates for d_e:

$$\frac{d_e}{e} \sim \frac{\alpha m_e}{M_{SUSY}^2} \left(\frac{\mathrm{Im}\, c_1}{a_1 a_7} + \frac{\mathrm{Im}\, c_3}{a_1 a_7} \frac{M_R \Delta m_{21}}{v_u^2} - \frac{b_1 \mathrm{Re}\, c_3}{a_1^2 a_7} \frac{m_\tau^2}{v_d^2} \left(\frac{M_R \Delta m_{21}}{v_u^2} \right)^2 + \ldots \right)^{11}$$

Flavor-blind → $M_{SUSY} \approx 500\, GeV$

Flavor-diagonal

Flavor off-diagonal
(≥ neutrino phases)

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(≥ neutrino phases)

$$\Delta m_{21} \approx \sqrt{\Delta m_\odot^2} \approx 10^{-9} - 10^{-11}\, GeV$$

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Flavor-blind $\gg$ Flavor-diagonal $\gg$ Flavor off-diagonal (≥ neutrino phases)

$$M_{SUSY} \approx 500\, GeV$$

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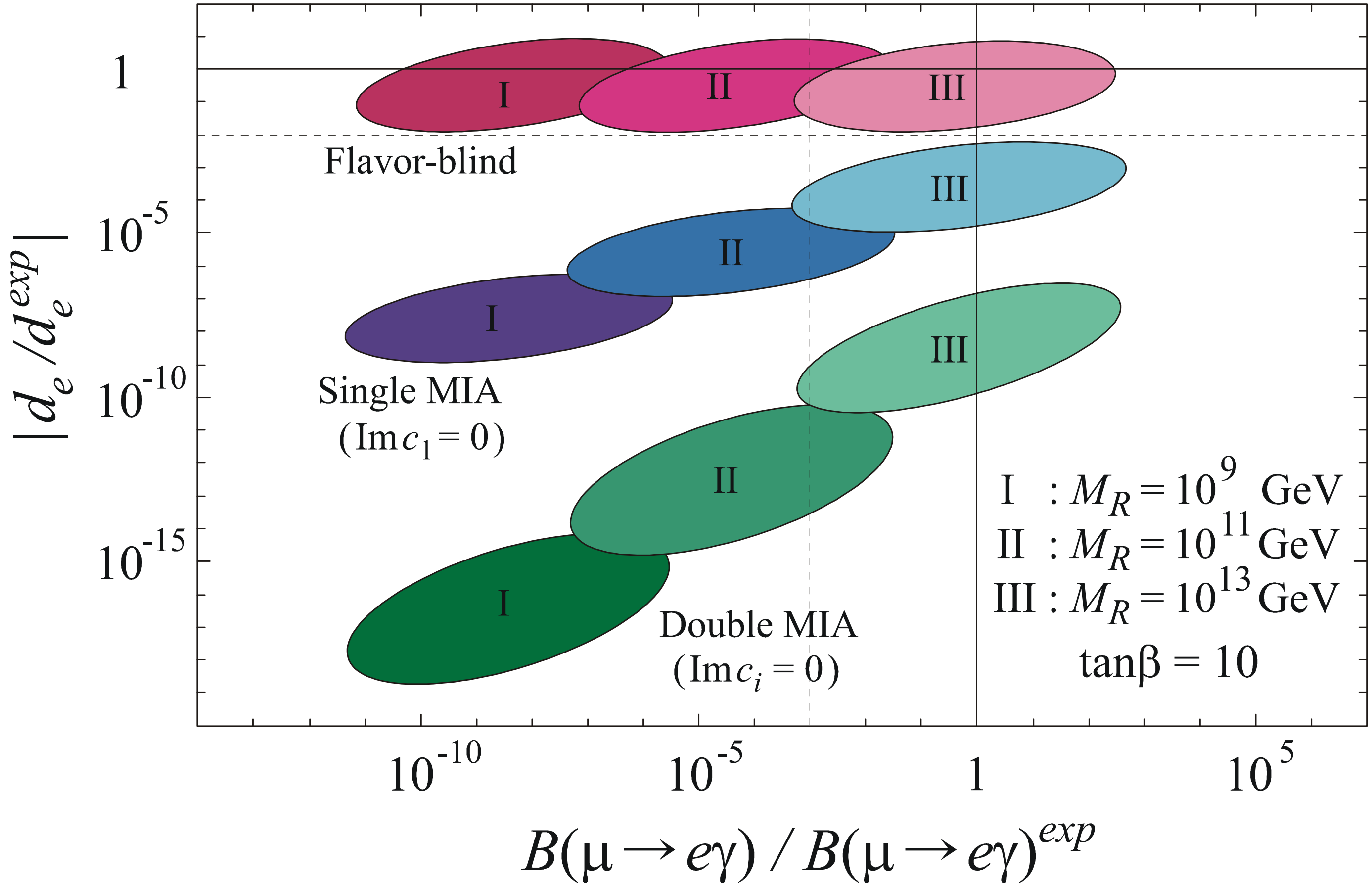
D. Impact on the EDMs and LFV processes

Mercolli,C.S. '09



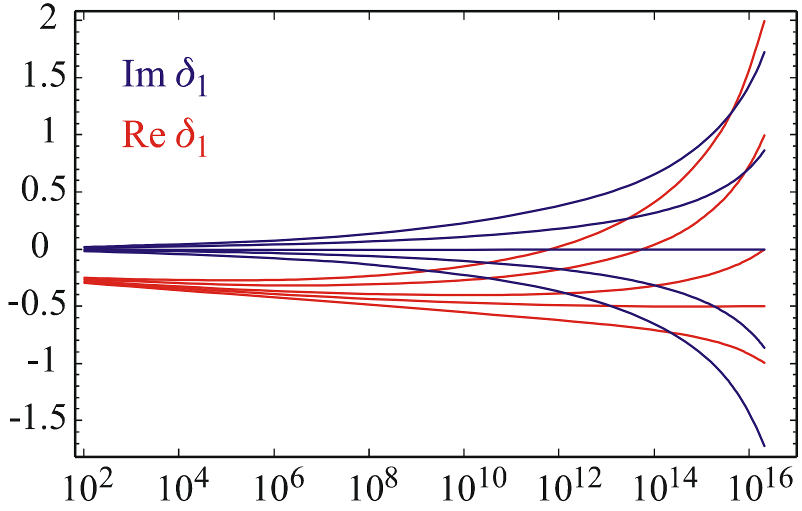
$$M_2 = \pm\mu = 2M_1 = \frac{2}{3}m_0 = A_0 = 400\,GeV\,,\ a_i, b_i, c_i, d_i \in \pm[0.1, 8]$$

III. RGE behavior

The MFV expansions are RGE invariant, but to get the RGE invariance of MFV itself requires in addition that the *coefficients must remain of $\mathcal{O}(1)$ at all scales*.

Running down from MFV at the GUT scale:

- *IR fixed-points* for ratios of coefficients ↔ predictions for *mass insertions.*
- In particular, all *CP-violating phases* run towards zero (in the quark sector).



$$\delta_1 \equiv \frac{(\delta^U_{RL})^{32}}{V_{ts}} = \frac{(\delta^U_{RL})^{31}}{V_{td}}$$

Paradisi,Ratz,Schieren,Simonetto '08
Colangelo,Nikolidakis,C.S. '08

Running up from MFV at the EW scale:

- MFV is lost at the GUT scale if one starts far enough from the fixed points.

(*some ratios of coefficients explode*)

IV. MFV instead of R-parity

A. MFV expansions and the flavor U(1) symmetries

Assume that the high-energy dynamics violates $\mathcal{B}$ and/or $\mathcal{L}$.

We want to parametrize the RPV couplings in terms of the spurions:

$$\mathcal{W}_{RPV} = \underbrace{\mu'^{I} L^I H_d + \lambda^{IJK} L^I L^J E^K + \lambda'^{IJK} L^I Q^J D^K}_{\Delta\mathcal{L}=1} + \underbrace{\lambda''^{IJK} U^I D^J D^K}_{\Delta\mathcal{B}=1}$$

Odd number of flavor indices → *MFV under* $SU(3)^5$ instead of $U(3)^5$, and use *ε–tensors* to form invariants.

Expected since $\mathcal{B}$ *and* $\mathcal{L}$ are combinations of the flavor $U(1)$'s:

$$\begin{aligned} G_f &= SU(3)^5 \times U(1)_Q \times U(1)_U \times U(1)_D \times U(1)_L \times U(1)_E \\ &= SU(3)^5 \times U(1)_{\mathcal{B}} \times U(1)_{\mathcal{L}} \times U(1)_Y \times U(1)_{PQ} \times U(1)_E \end{aligned}$$

But note: It is not needed to break all five $U(1)$'s!

B. Intrinsic difference between $\Delta\mathcal{L}=1$ and $\Delta\mathcal{B}=1$ couplings

- The $\mathcal{B}$ *violating couplings can be constructed* using $\Delta\mathcal{B}=0$ quark Yukawas:

$$\lambda''^{IJK} = \varepsilon^{LJK}(\mathrm{Y}_u\mathrm{Y}_d^\dagger)^{IL} \quad \Rightarrow \lambda''^{IJK}U^I D^J D^K \to \det(g_D^\dagger)\lambda''^{IJK}U^I D^J D^K$$
$$\lambda''^{IJK} = \varepsilon^{LMN}\mathrm{Y}_u^{IL}\mathrm{Y}_d^{JM}\mathrm{Y}_d^{KN} \quad \Rightarrow \lambda''^{IJK}U^I D^J D^K \to \det(g_Q^\dagger)\lambda''^{IJK}U^I D^J D^K$$
$$\cdots$$

- But $\mathcal{L}$ *violating couplings are strictly forbidden* as long as $m_\nu = 0$:

The SU(3) combinatorics demand a spurion transforming like a six.

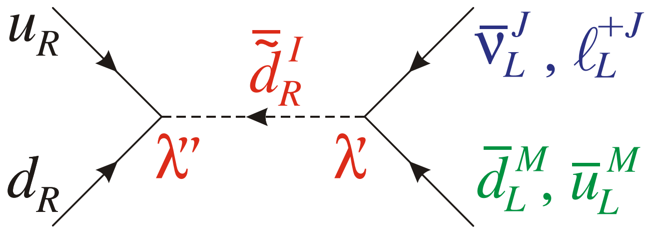
The only spurion available is the suppressed $\Delta\mathcal{L}=2$ Majorana mass term:

$$\Upsilon_\nu \equiv v_u \mathrm{Y}_\nu^T \mathrm{M}^{-1}\mathrm{Y}_\nu \to g_L^* \Upsilon_\nu g_L^\dagger \qquad \sim 6_{SU(3)_L} \otimes 1_{SU(3)_E}$$

All $\Delta\mathcal{L}=1$ couplings are suppressed by neutrino masses!!!

C. Proton decay without R-parity, but with MFV

Nikolidakis, C.S. '07



$$\Rightarrow \left|\lambda'_{JMI}\lambda''^{*}_{11I}\right| < 10^{-26}\left(m[\tilde{d}_R^I]/300\,GeV\right)^2$$

If the leading operators are: $\lambda' \; : \; (a_0 \varepsilon^{LMI} (\mathbf{Y}_e^\dagger \mathbf{Y}_e \Upsilon_\nu^\dagger)^{LM} (\mathbf{Y}_d \mathbf{Y}_u^\dagger \mathbf{Y}_u)^{KJ}) \; L^I Q^J D^K$

$$\lambda'' \; : \; (a_1 \varepsilon^{LJK} (\mathbf{Y}_u \mathbf{Y}_d^\dagger)^{IL} + a_2 \varepsilon^{LMN} \mathbf{Y}_u^{IL} \mathbf{Y}_d^{JM} \mathbf{Y}_d^{KN}) \; U^I D^J D^K$$

The MFV prediction is then

$$\left|\lambda'_{JMI}\lambda''^{*}_{11I}\right| \approx \frac{\Delta m_{31}}{v_u}\frac{m_\tau^2}{v_d^2} \; \lambda^3 \; \frac{m_b m_t^2 m_u}{v_d v_u^3} \left(a_0 a_1 \frac{m_s}{v_d} + a_0 a_2 \frac{m_d m_b}{v_d^2} \right)$$

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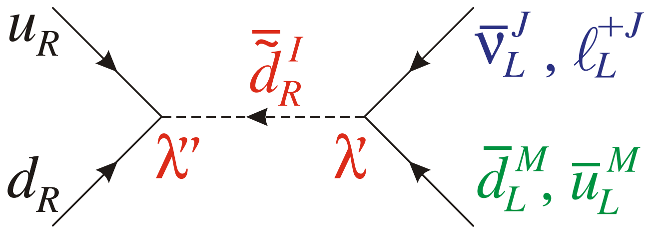

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Neutrino mass, related to $\Delta m_{atm}^2 \approx 3\cdot 10^{-3}\,\mathrm{eV}^2$

Charged lepton mass

CKM factors

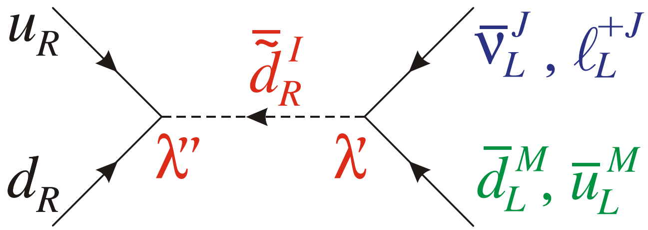
Light-quark masses

Symmetry of Υ_ν, the Majorana mass term

Antisymmetry of ε tensors

C. Proton decay without R-parity, but with MFV

Nikolidakis, C.S. '07



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$$\lambda'' \;:\; (a_1\varepsilon^{LJK}(\Upsilon_u \Upsilon_d^\dagger)^{IL} + a_2\varepsilon^{LMN}\Upsilon_u^{IL}\Upsilon_d^{JM}\Upsilon_d^{KN})\; U^I D^J D^K$$

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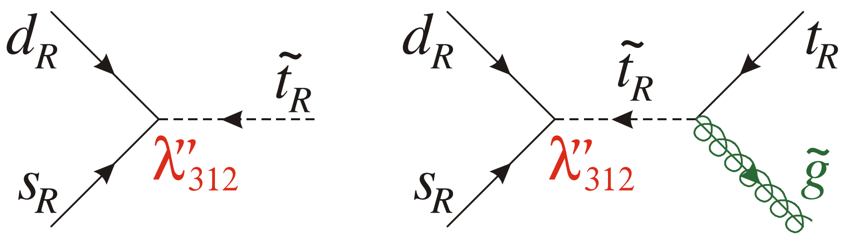
$$\approx a_0 a_1 10^{-28}\tan^4\beta + a_0 a_2 10^{-31}\tan^5\beta \qquad (for\; m_\nu^{lightest} = 0)$$

$\Rightarrow$ *MFV alone is sufficient to satisfy all the bounds on the proton lifetime.*

D. Possible signals at colliders

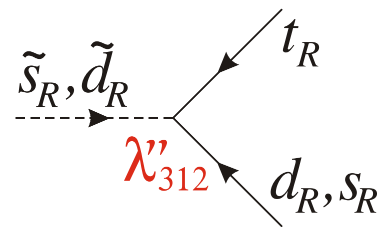
The only significant coupling is $\underset{(t\,d\,s)}{\lambda''_{312}} \sim 10^{-1}$ → Small effects in FCNC, Lepton number ~ conserved.

- *Single stop* resonant production and associated *single gluino* production:



Dimopoulos,Hall '88,Dreiner,Ross '91,...

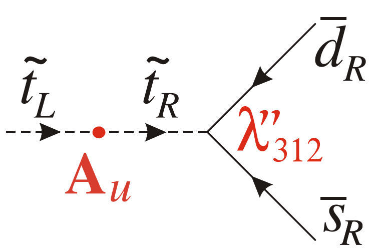
- *Top production* from down and strange squark decay:



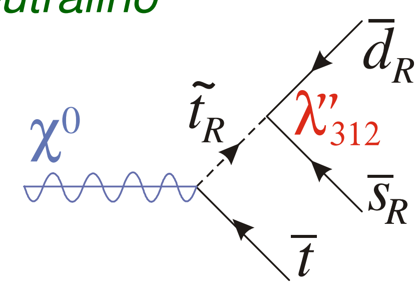
Berger et al. '99,Chiappetta et al. '99,...

- *LSP* not necessarily colorless & neutral, and will *decay*, maybe in the detector:

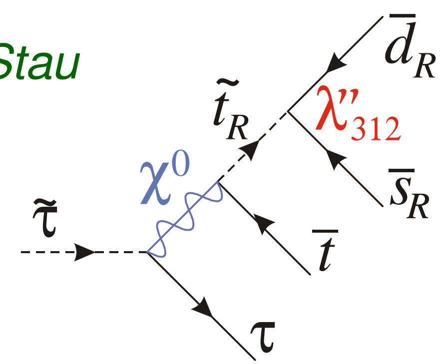
Stop



Neutralino



Stau



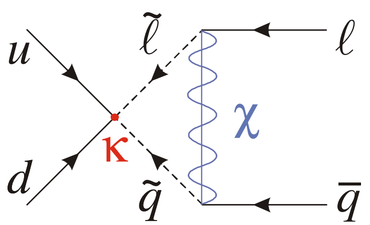
For a review of these and other possible signals, see e.g. Barbier et al. '05.

E. R-parity or not R-parity ?

- *Avoiding proton decay* is no longer a good motivation for R-parity.

- *Dim-5 R-parity conserving operators* can also induce proton decay:

Ibanez, Ross '92



$$\mathcal{W}_{\text{dim}-5} \ni \frac{\kappa_1^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}}(Q^I Q^J)(Q^K L^L) + \frac{\kappa_2^{IJKL}}{\Lambda_{\Delta\mathcal{L}=1}}(D^I U^J U^K)E^L$$

MFV separately suppresses $\Delta\mathcal{L} = 1$ and $\Delta\mathcal{B} = 1$ effects.

- *GUT*: R-parity often built in ($SO(10)$-GUT) or required ($SU(5)$-GUT).

Example: $G_f = U(3)_{\bar{5}} \times U(3)_{10} \ : \ \mathrm{Y}_{\bar{5}} \sim (\bar{3}, \bar{3}), \mathrm{Y}_{10} \sim (1, \bar{6})$

Seesaw spurion not required for $\mathcal{W}_{RPV} = \Lambda^{IJK} \bar{5}^I \bar{5}^J 10^K + \ldots$

Cirigliano, Grinstein, Isidori, Wise '05

- *Cosmology*: MSSM-LSP not stable → nature of dark matter still to be resolved.

Baryon asymmetry generated from MFV $\Delta\mathcal{B} = 1$ couplings?

Should experimentalists accept the burden of R-parity "only" for dark matter???

V. How to test MFV?

A. Back to the general MSSM (with R-parity)

Mercolli, C.S. '09

Generically, all flavor couplings expanded under MFV involve:

$$Q = x_1 1 + x_2 \mathrm{A} + x_3 \mathrm{B} + x_4 \mathrm{B}^2 + x_5 \{\mathrm{A},\mathrm{B}\} + x_6 \mathrm{BAB} + x_7 i[\mathrm{A},\mathrm{B}] + x_8 i[\mathrm{A},\mathrm{B}^2] + x_9 i(\mathrm{BAB}^2 - \mathrm{B}^2\mathrm{AB})$$

$$(\mathrm{A} \equiv \mathrm{Y}_e^\dagger \mathrm{Y}_e, \mathrm{B} \equiv \mathrm{Y}_\nu^\dagger \mathrm{Y}_\nu)$$

$$(\mathrm{A} \equiv \mathrm{Y}_d^\dagger \mathrm{Y}_d, \mathrm{B} \equiv \mathrm{Y}_u^\dagger \mathrm{Y}_u)$$

The MFV operators form a *complete basis* for soft-breaking terms.

Allowing the coefficients to take any value → *full MSSM*.

MFV expansion coefficients versus Mass Insertions:

Same number of free parameters (choice of basis).

BUT: to each coefficient corresponds a *whole set of mass insertions*, with a *definite flavor pattern*, inherited from those of the spurions.

↓

Permits to *test the naturality* of soft-breaking terms.

B. Experimental constraints on the generic MSSM slepton sector

m_L^2	(x_i / a_1)	m_R^2	(x_i / a_7)	$\text{Re}\, A_e$	$(x_i / a_1 a_7)$	$\text{Im}\, A_e$	$(x_i / a_1 a_7)$
a_1	*free*	a_7	*free*	$\text{Re}\, c_1 \le 10^2$	*stab.*	$\text{Im}\, c_1 \le 2$	d_e
$a_2 \le 10^3$	*masses*	$a_8 \le 10^3$	*masses*	$\text{Re}\, c_2 \le 10^3$	*stab.*	$\text{Im}\, c_2 \le 10^3$	*stab.*
$a_3 \le 10$	$\mu \to e\gamma$	$a_9 \le 10^6$	$\tau \to \mu\gamma$	$\text{Re}\, c_3 \le 10^3$	$\mu \to e\gamma$	$\text{Im}\, c_3 \le 10^3$	$\mu \to e\gamma$
$a_4 \le 10^4$	$\mu \to e\gamma$	$a_{10} \le 10^9$	$\tau \to \mu\gamma$	$\text{Re}\, c_4 \le 10^6$	$\mu \to e\gamma$	$\text{Im}\, c_4 \le 10^6$	$\mu \to e\gamma$
$a_5 \le 10^3$	$\tau \to \mu\gamma$	$a_{11} \le 10^7$	$\tau \to \mu\gamma$	$\text{Re}\, c_5 \le 10^5$	$\tau \to \mu\gamma$	$\text{Im}\, c_5 \le 10^5$	$\tau \to \mu\gamma$
$a_6 \le 10^4$	$\mu \to e\gamma$	$a_{12} \le 10^{11}$	$\tau \to \mu\gamma$	$\text{Re}\, c_6 \le 10^7$	$\mu \to e\gamma$	$\text{Im}\, c_6 \le 10^7$	$\mu \to e\gamma$
$b_1 \le 10^3$	$\tau \to \mu\gamma$	$b_4 \le 10^7$	$\tau \to \mu\gamma$	$\text{Re}\, d_1 \le 10^5$	$\tau \to \mu\gamma$	$\text{Im}\, d_1 \le 10^5$	$\tau \to \mu\gamma$
$b_2 \le 10^6$	$\tau \to \mu\gamma$	$b_5 \le 10^{10}$	$\tau \to \mu\gamma$	$\text{Re}\, d_2 \le 10^8$	$\tau \to \mu\gamma$	$\text{Im}\, d_2 \le 10^8$	$\tau \to \mu\gamma$
$b_3 \le 10^8$	$\mu \to e\gamma$	$b_6 \le 10^{13}$	$\tau \to \mu\gamma$	$\text{Re}\, d_3 \le 10^{10}$	$\mu \to e\gamma$	$\text{Im}\, d_3 \le 10^{10}$	$\mu \to e\gamma$

$M_{SUSY} \approx 500\, GeV,\ \tan\beta = 20,\ M_R = 10^{12}\, GeV, m_{L,R} \le 4\,\text{TeV}$

Compared to MIA: If a coefficient must be $x_i \gg 1 \Rightarrow$ *New flavor structures*

If a coefficient must be $x_i \ll 1 \Rightarrow$ *Fine-tuning problem is back*

If all coefficients are $x_i \approx 1 \Rightarrow$ *MFV*

Conclusion

MFV, as a phenomenological hypothesis on the elementary flavor structures:

A single mechanism explaining: - *Smallness of susy effects in FCNC*
- *Extremely long proton lifetime*

Consequences of the *Yukawa hierarchies* and of the *small neutrino masses*.

MFV, as a window into physics beyond the MSSM:

It permits to *identify the flavor couplings which are fine-tuned* (none at present) out of those which are as "natural" as the SM Yukawas.

In particular, *the proton lifetime does not require fine-tuned RPV couplings*!

Since a *consistent picture emerges with only a few spurions*, the mechanism behind all the flavor structures could be relatively simple.

CP-violation is controlled by non-MFV physics, as expected from $Arg(\mu) \ll 1$.